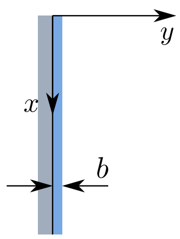
MEC-E1020 Fluid dynamics - Exam - 22.2.2019

The general guidelines for assessment are specified for each task below. These guidelines will be adapted on a case by case basis. In order to pass, you need to get at least 8 points in total.

1 Fundamental equations and their solutions

Let's consider a thin film of water flowing down a vertical wall. The thickness b of the film is constant. The pressure is constant everywhere and the velocity field is given by

$$u(y) = \frac{gby}{\nu}\left(1 - \frac{y}{2b}\right), \quad v = 0, \quad w = 0 \, .$$



a) Calculate the force per unit area acting on the wall. (1p)

b) Show that the volume of a differential fluid element does not change in this case. (1p)

c) Discuss the balance of forces acting on a differential fluid element in this case. (1p)

d) Discuss the balance of energy in this case and support your claims by appropriate analysis of the balance of the mechanical energy

$$\rho\frac{D}{Dt}\int_V \frac{1}{2}u_i^2 dV = \int_V \rho g_i u_i dV + \int_A u_i \tau_{ij} n_j dA + \int_V p\frac{\partial u_i}{\partial x_i} dV - \int_V 2\mu e_{ij} e_{ij} dV \, . \text{(3p)}$$

Assessment

- Have a look at sections 3.6, 4.5, 4.7, 4.10 and 4.13 as well as the exercise for week 1.

Subtask a

- The shear stress on the wall is evaluated based on the stress-strain relationship (+0.5) and the force per unit area is evaluated correctly (+0.5).

Subtask b

- The rate of change of the volume is related to the divergence of the velocity field (+0.5), which is shown to be zero for the given field (+0.5).

Subtask c

- The balance of the net shear force and the gravitational force is sensibly discussed. +1

Subtask d

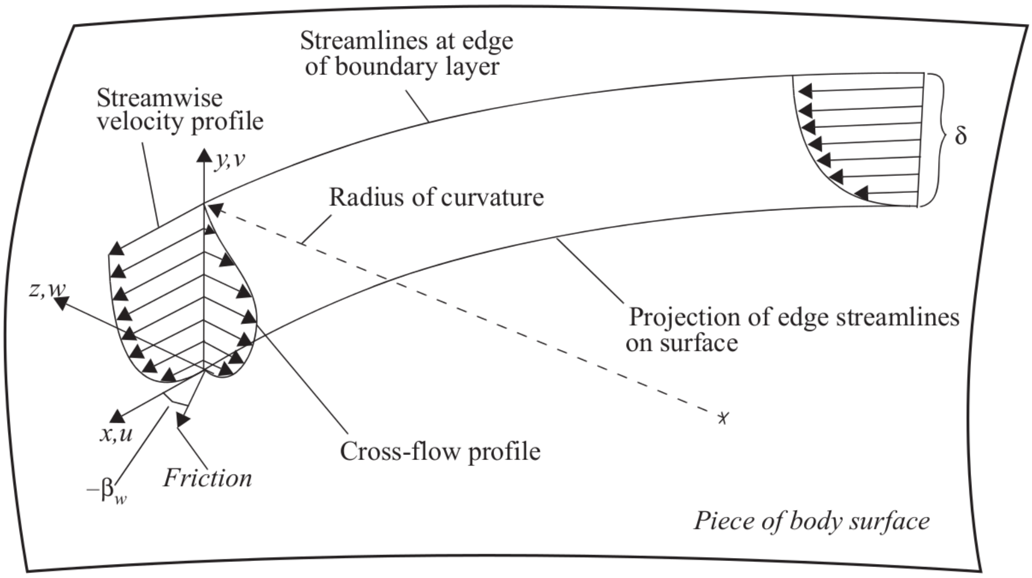
- The left (+0.5) and the right (+0.5) hand sides of the mechanical energy equation are simplified correctly and simplifications are justified (+1).
- The balance of the rate of work done by the gravity and the viscous dissipation of heat is treated sensibly. +1

2 Boundary layers and related flows

a) It can be shown that within a boundary layer some of the terms in the full Navier-Stokes equations can be neglected. How can this be justified? How does this change the way in which the equations can be solved? (2p)

- Have a look at section 10.2 in Kundu and Cohen and the order of magnitude analysis in slide set 3.2 and slide 25 in set 3.1.
- The orders of magnitude of the different terms are considered in the answer (+0.5) and these are linked to the characteristics of boundary layers (+0.5).
- The influence on the solution procedure is sensibly discussed. +1

b) Let's study a three dimensional flow within a boundary layer. Explain, how the cross-flow in the picture on the next page is generated. (2p)



- Have a look at section 10.13 in Kundu and Cohen and slide 16 in set 3.1.
- The answer considers the variation of the pressure through the boundary layer and the influence of the transversal pressure gradient (+1) as well as the influence of the variation of the velocity (+1) on the curvature of the streamlines.

c) Explain, why the boundary layer developing on the surface of a body is displacing the streamlines away from the body. How can you evaluate the size of this displacement, if you know the velocity profile within the boundary layer? (2p)

- Have a look at section 10.3 in Kundu and Cohen.
- The displacement effect has been explained sensibly based on the mass conservation. +1
- The size of the displacement has been given using the integral formula for the displacement thickness. +1

3 Instability and turbulence

a) Compare laminar and turbulent boundary layers in terms of the evolution of the boundary layer thickness, the shape of the mean velocity profile, the nondimensional shear stress on the wall and flow separation. What explains the differences in these characteristics? (2p)

- Have a look at sections 10.8 and 13.1 in Kundu and Cohen and slide 15 in slide set 4.1.
- The answer highlights the differences in the named characteristics between laminar and turbulent cases. +4*0.25
- The answer considers the different mechanism for momentum transfer in laminar and turbulent flows. +1

b) What do we mean by the Reynolds stress and how is it typically modelled? (2p)

- Have a look at sections 13.5 and 13.12 in Kundu and Cohen and slide 22 in slide set 4.1.
- The link to the transfer of momentum due to turbulent motions is sensibly discussed. +1
- The modelling based on the concept of eddy viscosity is briefly explained. +1

c) What do we mean by vortex stretching in the context of turbulent flows and how is this relevant in terms of the energy balance of turbulent flows? (2p)

- Have a look at section 13.8 in Kundu and Cohen.
- The stretching/contraction of a vortex element and the influence of this on the strength of the vortex element is reasonably discussed. +1
- The process of energy transfer from the mean flow to the largest eddies (+0.5) and between the eddies from larger to smaller ones (+0.5) is reasonably discussed.

4 Numerical techniques

A discretised form of the one-dimensional convection-diffusion equation is given as

$$T_i^n + \alpha\left(T_i^n - T_{i-1}^n\right) - \beta\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) = T_i^{n-1}$$

$$\alpha = \frac{u\Delta t}{\Delta x}\,, \quad \beta = \frac{D\Delta t}{\Delta x^2}\,,$$

where u is assumed to be a positive velocity and D a diffusion coefficient. The subscripts refer to different points of the spatial discretisation and the superscripts to the discrete time levels.

a) Is this an explicit or implicit scheme? Justify your answer. (1p)

b) Calculate the truncation error in this case. What is the spatial and temporal order of accuracy? (3p)

c) Identify the discretisation schemes (backward, forward, centered) used in the discretised equation. How would you change the schemes, if you would like to improve the accuracy of the spatial discretisation without reducing the grid spacing? (2p)

Assessment

- Have a look at section 11.2 in Kundu and Cohen.

Subtask a

- The type of the scheme (implicit) is correctly identified and the answer is justified based on the number of unknowns in the discretised equation. +1

Subtask b

- Taylor series expansion is applied correctly for the previous, left neighbour and right neighbour values. +3*0.5
- Taylor series expansion is substituted into the discrete equation and equation cleaned up to reveal the truncation error. +0.5
- Truncation error is correct. +0.5
- The order of the scheme in space and time (1st in both) is correctly identified. +2*0.25

Subtask c

- The schemes are correctly identified: backward for time derivative, backward for convective derivative, centered for diffusion term. +3*0.5
- Sensible improvement suggested (e.g. change the scheme for convective term from backward to centered). +0.5