Sallittu oheismateriaali: taskulaskin (myös ohjelmoitavat ja graafiset laskimet käyvät) ja oma, ohjeiden mukainen kaavakokoelma.

Palauta vähintään yksi nimelläsi varustettu konsepti. Muista palauttaa myös monivalintatehtäväpaperi. Palauta kaikki saamasi yliopiston konseptiarkit – myös tyhjät ja suttupaperit. Tämän tehtäväpaperin ja oman kaavakokoelmasi voit pitää.

1. Monivalintatehtävä erillisellä paperilla.

2. Varaus $Q > 0$ sijaitsee xy-tason pisteessä (a, a).

(a) Laske sähkökentän voimakkuus x-akselilla pisteessä $(x, 0)$.

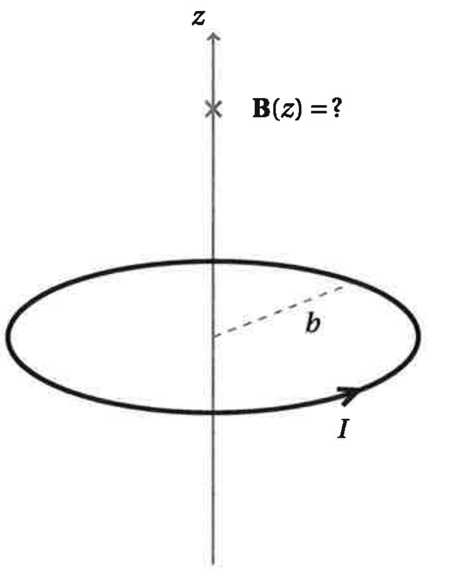
(b) Varausta $Q_t > 0$ kuljetetaan origosta alkaen x-akselia pitkin äärettömän kauas. Onko työ positiivinen vai negatiivinen?

(c) Laske (b)-kohdan työ määritelmästä $W = \int \mathbf{F} \cdot d\boldsymbol{\ell}$ lähtien.

LASKE 3. TEHTÄVÄ **ERI KONSEPTILLE KUIN** 2. TEHTÄVÄ!

3. Laske Biot–Savartin lain perusteella b-säteisen tasavirtasilmukan aiheuttaman magneettivuon tiheys $\mathbf{B}$ symmetria-akselilla z. Sijaitkoon silmukka origokeskisesti xy-tasolla, joten laske magneettivuontiheysvektorifunktio kaikilla z:n arvoilla $-\infty < z < \infty$, kun $x = 0, y = 0$.

Vuontiheyden lauseke yksinkertaistuu, kun ollaan kaukana silmukasta, siis kun $z \gg b$. Mitä se on tällöin?

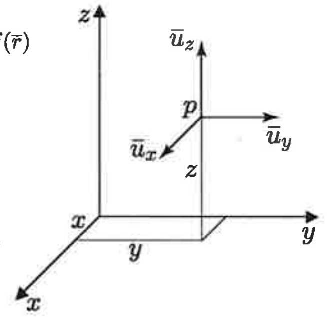


Muistat varmaan Biot–Savartin lain:

$$d\mathbf{H} = \frac{I\, d\boldsymbol{\ell} \times \mathbf{u}_r}{4\pi r^2}$$

Nablaoperaatiot

Karteesinen koordinaatisto



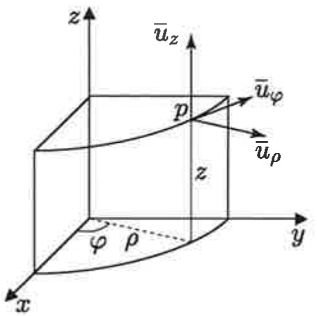
$$\nabla f(\overline{r}) = \overline{u}_x \frac{\partial}{\partial x} f(\overline{r}) + \overline{u}_y \frac{\partial}{\partial y} f(\overline{r}) + \overline{u}_z \frac{\partial}{\partial z} f(\overline{r})$$

$$\nabla \times \overline{f} = \begin{vmatrix} \overline{u}_x & \overline{u}_y & \overline{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f}(\overline{r}) = \frac{\partial}{\partial x} f_x(\overline{r}) + \frac{\partial}{\partial y} f_y(\overline{r}) + \frac{\partial}{\partial z} f_z(\overline{r})$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Sylinterikoordinaatisto



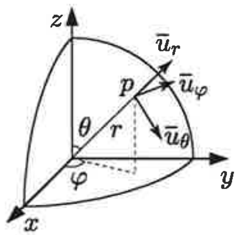
$$\nabla f(\overline{r}) = \overline{u}_\rho \frac{\partial}{\partial \rho} f + \overline{u}_\varphi \frac{1}{\rho} \frac{\partial}{\partial \varphi} f + \overline{u}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \overline{f} = \frac{1}{\rho} \begin{vmatrix} \overline{u}_\rho & \rho \overline{u}_\varphi & \overline{u}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ f_\rho & \rho f_\varphi & f_z \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho f_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \varphi} f_\varphi + \frac{\partial}{\partial z} f_z$$

$$\nabla^2 f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Pallokoordinaatisto



$$\nabla f(\overline{r}) = \overline{u}_r \frac{\partial}{\partial r} f + \overline{u}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} f + \overline{u}_\varphi \frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} f$$

$$\nabla \times \overline{f} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \overline{u}_r & r\overline{u}_\theta & r\sin\theta \overline{u}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ f_r & r f_\theta & r \sin\theta f_\varphi \end{vmatrix}$$

$$\nabla \cdot \overline{f} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 f_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta f_\theta) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} f_\varphi$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 f}{\partial \varphi^2}$$

Koordinaattimuunnokset vektorille $\overline{f}$

Karteesinen ↔ sylinterikoordinaatisto

$x = \rho \cos\varphi, \quad y = \rho \sin\varphi, \quad z = z,$

$\rho = \sqrt{x^2 + y^2}, \quad \varphi = \arctan(y/x), \quad z = z.$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix},$$

$$\begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

Karteesinen ↔ pallokoordinaatisto

$x = r \sin\theta \cos\varphi, \quad y = r \sin\theta \sin\varphi, \quad z = r \cos\theta,$

$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \arctan\left(\frac{\sqrt{x^2 + y^2}}{z} \right), \quad \varphi = \tan^{-1} \frac{y}{x}$

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} f_r \\ f_\theta \\ f_\varphi \end{pmatrix},$$

$$\begin{pmatrix} f_r \\ f_\theta \\ f_\varphi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}.$$

Sylinteri ↔ pallokoordinaatisto

$\rho = r \sin\theta, \quad \varphi = \varphi, \quad z = r\cos\theta,$

$r = \sqrt{\rho^2 + z^2}, \quad \theta = \arctan(\rho/z), \quad \varphi = \varphi.$

$$\begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix} = \begin{pmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{pmatrix} \begin{pmatrix} f_r \\ f_\theta \\ f_\varphi \end{pmatrix},$$

$$\begin{pmatrix} f_r \\ f_\theta \\ f_\varphi \end{pmatrix} = \begin{pmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix}.$$

Vektori-integraalilaskennan kaavoja

Karteesinen koordinaatisto

$\overline{d\ell} = \overline{u}_x\, dx + \overline{u}_y\, dy + \overline{u}_z\, dz$

$\overline{dS_x} = \overline{u}_x\, dy\, dz$

$\overline{dS_y} = \overline{u}_y\, dx\, dz$

$\overline{dS_z} = \overline{u}_z\, dx\, dy$

$dV = dx\, dy\, dz$

Sylinterikoordinaatisto

$\overline{d\ell} = \overline{u}_\rho\, d\rho + \overline{u}_\varphi \rho\, d\varphi + \overline{u}_z\, dz$

$\overline{dS_\rho} = \overline{u}_\rho \rho\, d\varphi\, dz$

$\overline{dS_\varphi} = \overline{u}_\varphi\, d\rho\, dz$

$\overline{dS_z} = \overline{u}_z \rho\, d\rho\, d\varphi$

$dV = \rho\, d\rho\, d\varphi\, dz$

Pallokoordinaatisto

$\overline{d\ell} = \overline{u}_r\, dr + \overline{u}_\theta r\, d\theta + \overline{u}_\varphi r \sin\theta\, d\varphi$

$\overline{dS_r} = \overline{u}_r r^2 \sin\theta\, d\theta\, d\varphi$

$\overline{dS_\theta} = \overline{u}_\theta r \sin\theta\, dr\, d\varphi$

$\overline{dS_\varphi} = \overline{u}_\varphi r\, dr\, d\theta$

$dV = r^2 \sin\theta\, dr\, d\theta\, d\varphi$

Gaussin lause $\int_V \nabla \cdot \overline{f}\, dV = \oint_S \overline{f} \cdot \overline{dS}$

Stokesin lause $\int_S \nabla \times \overline{f} \cdot \overline{dS} = \oint_C \overline{f} \cdot \overline{d\ell}$

Vakioita

$\epsilon_0 = 8.854 \cdot 10^{-12} \, \frac{\mathrm{As}}{\mathrm{Vm}}$

$\mu_0 = 4\pi \cdot 10^{-7} \, \frac{\mathrm{Vs}}{\mathrm{Am}}$

$k_B = 1.38 \cdot 10^{-23} \, \frac{\mathrm{J}}{\mathrm{K}}$

$e = 1.60 \cdot 10^{-19} \, \mathrm{C}$