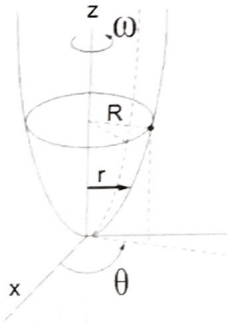
Aalto University
School of Science

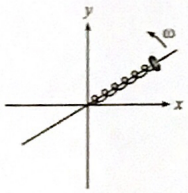
PHYS-E0419 - Dynamics of particles, fluids and solids
Midterm exam Friday 25.10.2019

1. Explain following concepts biefly (max 1p each)
   (a) Holonomic constraint
   (b) Virtual displacement
   (c) Action
   (d) Hamiltonian and its relation to Lagrangian
   (e) Cyclic coordinate
   (f) What does it mean if we say the lagrangian has a symmetry?

2. a) A bead of mass m slides under the influence of gravity along a parabolic wire where $z = cr^2$. The wire rotates with angular velocity ω about the vertical axis. Obtain the system's Lagrangian using radial coordinate (of the cylindrical coordinate system) as the genereralized coordinate. Identify canonical momentum. (2p.)
   b) Derive the system's Lagrange equation (2p.)
   c) How fast should the wire rotate in order to suspend the bead at an equilibrium at height $z > 0$. (2p.)



3. a) A bead of mass m moves along a massless rod that is pivoted at the origin and arranged (via an external torque) to rotate with a potentially time dependent angular speed $\omega = f(t)$. A spring with spring constant k and relaxed length of zero lies along the rod and connects the mass to the origin. Construct the lagrangian and hamiltonian for the system using polar coordinate r as the generalized coordinate. (Note: there is no gravity in the problem.) (4p.)
   b) Show that if ω is constant Hamiltonian is conserved. (2p.)



4. a) Derive Lagrange's equation using the Hamilton's priciple. (It is enough to do so for a system with one generalized coordinate.) (4p.)
   b) Show that constructing a new lagrangian L' from earlier one $L(q, \dot{q}, t)$ by

$$L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \frac{dF(q, t)}{dt}, \tag{1}$$

   gives rise to same equations of motion. $F(q, t)$ is some function of coordinate(s) and times. (2p)

Remember to answer in english unless you have a special permission to use some other language. Write your name, student number, study program, course code, and the date in all your papers. Use of calculators is forbidden.