MS-E2114 Investment Science

Examination 19.12.2018 **Gustafsson**

Please write the following information on each answer sheet:

- MS-E2114 Investment Science
- student registration number, surname and all first names
- study programme and year of studies
- signature

A standard calculator is the only permitted aid in this examination.

1. Please briefly explain the following concepts:
   a) Internal rate of return (1p)
   b) Short selling (1p)
   c) Jensen's index (1p)
   d) Delta of an option (1p)
   e) Minimum variance hedge (1p)
   f) Geometric Brownian motion (1p)

2. Please explain the following topics relating to portfolio theory:
   a) Objectives of the investor in the Markowitz model (1p)
   b) Efficient frontier (1p)
   c) Capital market line (1p)
   d) Security market line (1p)

3. Consider a one-period setting with (i) a risk-free asset with a risk-free interest rate of 2% p.a. and (ii) a non-dividend-paying stock whose price follows a binomial lattice. The lattice starts with an initial state at time 0 and ends with states u and d at time 1 (measured in years). The current price of the stock is 80 euro. In state u, the stock price is multiplied by 1.1 and in state d by 1/1.1. The probability that state u occurs is 75%. The investor can borrow and lend at the risk-free interest rate without limit. Also, the stock can be bought and sold (including short-sold) without limit. Both the risk-free asset and the stock can be bought and sold in fractional quantities.
   a) What is the forward price of the stock at time 1 (in one year from present)? Justify your answer. (1p)
   b) Construct a portfolio ("a synthetic security") using the risk-free asset and the stock so that the portfolio yields 1 euro in state u and 0 euro in state d. What is the present value of this portfolio? What are the positions of the risk-free asset and the stock in this portfolio? (1p)
   c) Construct another such portfolio using the risk-free asset and the stock so that the portfolio yields 0 euro in state u and 1 euro in state d. What is the present value of this portfolio? What are the positions of the risk-free asset and the stock in this portfolio? (1p)
   d) Consider a European call option on the stock with a strike price of 85 euro and expiration at time 1. Calculate the cash flows of this option in states u and d. (1p)
   e) Use the synthetic securities in (b) and (c) to determine the present value (price) of the call option. (1p)

f) What is the replicating portfolio for the call option? What are the positions of the risk-free asset and the stock in this portfolio? (1p)

4. Suppose that there are only two stocks in the market portfolio, stocks A and B, which do not pay dividends and which are *uncorrelated* with each other. They have the following characteristics:

Stock	Number of outstanding shares	Share price	Standard deviation of rate of return
A	100	€5.00	20%
B	200	€3.00	30%

The expected rate of return of the market portfolio is 12% and the risk-free interest rate is 2%. Suppose also that the assumptions of the Capital Asset Pricing Model (CAPM) hold.

a) Determine the betas for stocks A and B. (2p)
b) Determine the expected rates of return for stocks A and B. (1p)

Consider 1-year European call and put options on stock A as a function of strike price K. Justify your answers.

c) Does the price of a call option increase or decrease as K increases? (0.5p)
d) Does the price of a put option increase or decrease as K increases? (0.5p)
e) What is the price of a call when $K = 0$? Note that the stock does not pay dividends. (0.5p)
f) What is the price of a put when $K = 0$? (0.5p)
g) What is the limit value of the price of a call as K approaches infinity? (0.5p)
h) What is the limit value of the price of a put as K approaches infinity? (0.5p)
i) Determine the strike price K^* at which a 1-year European call option and a 1-year European put option on stock A have exactly the same price. (2p)

5. A company has three liabilities whose maturities are 1, 2, and 4 years. The liabilities and respective spot rates (using annual compounding) are as follows:

Year	1	2	3	4
Liability (million €)	1	1	-	3
Spot rate	2%	3%	3%	4%

a) Determine the present value and duration of the liabilities. (2p)
b) The company seeks protection from possible parallel shifts in spot rates by investing in an immunizing portfolio consisting of bonds A and B. Bond A has a coupon rate of 1%, matures in 2 years, and pays coupons annually. Bond B is a zero-coupon bond with maturity in 4 years. How much must the company invest in bonds A and B, respectively? (4p)