FINAL EXAM, MATRIX ALGEBRA, MS-A0011

- Time: 11.12.2018, 16:30–19:30
- Equipment: None
- Answer each problem on a separate sheet. Each problem is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points.
- Good luck, have fun! /Ragnar

Problem 1

Let A be a 3×3 matrix that has eigenvalues $0, 1, 2$. For each of the statements below, write if they are necessarily true, necessarily false, or if they can be either true or false.

a) The equation $A\mathbf{x} = \mathbf{x}$ has a solution.

b) The equation $A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ has a solution.

c) $|A| = 1$.

d) A is diagonalizable.

Problem 2

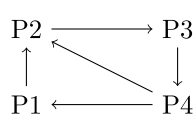
Diagonalize the matrix $\begin{pmatrix} 0 & -2 & 2 \\ 2 & -4 & 2 \\ 2 & -2 & 0 \end{pmatrix}$.

Problem 3

Find an orthonormal basis for the space $\text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \right\}$

Problem 4

Give the importance ranking for this web of pages, according to the PageRank algorithm (with $\alpha = 0$):



Problem 5

Let $r : \mathbb{R}^3 \to \mathbb{R}^3$ be the map given by

$$r(\mathbf{e}_1) = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2), r(\mathbf{e}_2) = \frac{1}{\sqrt{2}}(\mathbf{e}_2 - \mathbf{e}_1), \text{ and } r(\mathbf{e}_3) = \mathbf{e}_3.$$

Let $p : \mathbb{R}^3 \to \mathbb{R}^3$ be orthogonal projection to the plane

$$\{\mathbf{x} \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}.$$

Compute the transformation matrix of the composition $p \circ r$.