COURSE EXAM, MATRIX ALGEBRA, MS-A0011

- Time: 29.5.2019, 16:30–19:30
- Equipment: None
- Answer each problem on a separate page. Each problem is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points.
- Good luck, have fun! /Ragnar

Problem 1

Let A and B be two invertible $n \times n$ matrices, and let $\mathbf{v} \in \mathbb{R}^n$ be a vector that is an eigenvector of both A and B. For each of the statements below, write if they are necessarily true, necessarily false, or if they can be either true or false.

a) AB is invertible.
b) $A + B$ is invertible.
c) $\mathbf{v}$ is an eigenvector of AB.
d) $\mathbf{v}$ is an eigenvector of $A + B$.

Problem 2

Diagonalize the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 2 & -1 & -2 \\ 1 & -2 & 2 \end{pmatrix}$.

Problem 3

Let

$$V = \text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\},$$

and let $F(\mathbf{x})$ be the point on V that is closest to $\mathbf{x}$, for $\mathbf{x} \in \mathbb{R}^4$. Determine the transformation matrix of F.

Problem 4

Consider three points in Euclidean 3-dimensional space, with coordinates as follows:

$$P_1 : (1, -1, 1), P_2 : (-1, 2, 2), P_3 : (2, 1, -1).$$

Compute the area of the triangle that has corners in P_1, P_2, and P_3.

Problem 5

Compute an orthonormal basis for

$$\text{Span}\left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} \right\}.$$