

COURSE EXAM, MATRIX ALGEBRA, MS-A0011

- Time: 10.12.2019, 16:30–19:30
- Equipment: One sheet (A4) of hand-written notes, written on one side only.
- Answer each problem on a separate page. Each problem is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points.
- Good luck, have fun! /Ragnar

PROBLEM 1

Let A be a 3×3 matrix with determinant $|A| = 0$, let $\mathbf{v} \in \mathbb{R}^3$ be a non-zero column vector, and let I be the 3×3 identity matrix. For each of the statements below, write if they are necessarily true, necessarily false, or if they can be either true or false.

- The columns of A form a basis for \mathbb{R}^3 .
- $A + I$ is invertible.
- \mathbf{v} is an eigenvector of A .
- \mathbf{v} is an eigenvector of I .

PROBLEM 2

Diagonalize the matrix $\begin{pmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ 2 & -4 & 3 \end{pmatrix}$.

PROBLEM 3

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be given by

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3x + 2y \\ x + y + 2z \\ 4x + 3y + 2z \end{pmatrix}.$$

- What is the transformation matrix of T in the standard basis?
- Determine a basis for the null space of T .
- What is the rank of T ?

PROBLEM 4

Let L_1 be the line given by the equation $x = y = z$ in \mathbb{R}^3 , and let L_2 be the line through the points $(1, 1, 1)$ and $(-1, -2, c)$.

- For what value of c do the lines L_1 and L_2 intersect?
- For this value of c , determine an equation for the plane that contains both the lines.

PROBLEM 5

Let

$$L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{pmatrix}.$$

What is the smallest possible value of $\left\| L\mathbf{x} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\|$, when $\mathbf{x} \in \mathbb{R}^2$?