**Task 1**

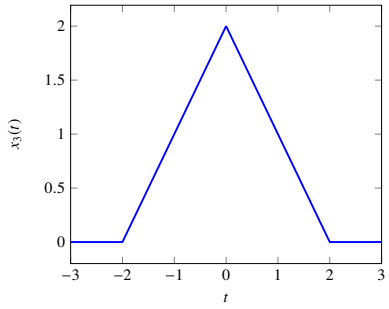
a) (2p.)

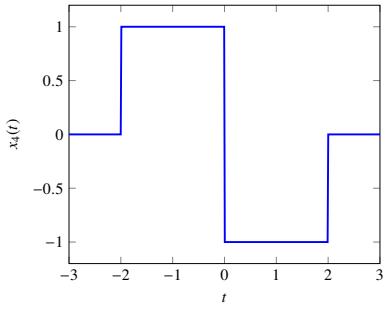
$$\langle \alpha x_1(t) - \beta x_2(t), x_1(t) \rangle = \alpha \langle x_1(t), x_1(t) \rangle - \beta \langle x_2(t), x_1(t) \rangle = \alpha |x_1(t)|^2 = \alpha$$

$$\langle \alpha x_1(t) - \beta x_2(t), x_2(t) \rangle = \alpha \langle x_1(t), x_2(t) \rangle - \beta \langle x_2(t), x_2(t) \rangle = -\beta |x_2(t)|^2 = -\beta$$

b) (2p.)

$$x_4(t) = \frac{d}{dt} 2 \cdot \text{tria}\left(\frac{t}{2}\right) = \text{rect}\left(\frac{t+1}{2}\right) - \text{rect}\left(\frac{t-1}{2}\right)$$

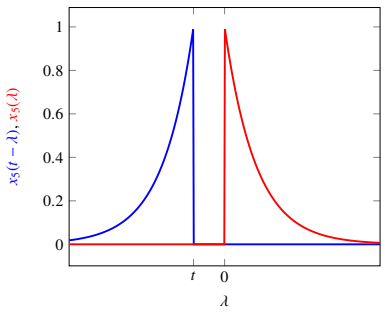


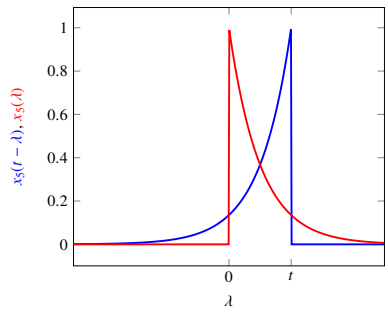




c) (2p.)

$$\int_{-\infty}^{\infty} x_3(t)\delta(t-1)dt = \int_{-\infty}^{\infty} x_3(1)\delta(t-1)dt = x_3(1)\int_{-\infty}^{\infty} \delta(t-1)dt = x_3(1) = 1$$

d) (4p.) .



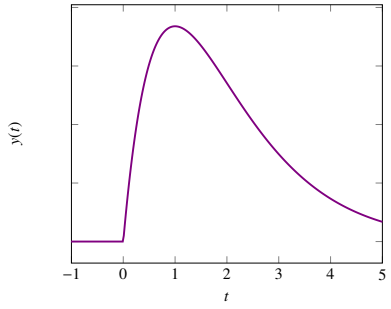

When $t \leq 0$ signals $x_5(\lambda)$ and $x_5(t-\lambda)$ do not overlap and so convolution $y(t) = (x_5 \otimes x_5)(t) = 0$.

When $t > 0$, pulses do overlap on interval $[0, t]$, and convolution can be calculated as follows:

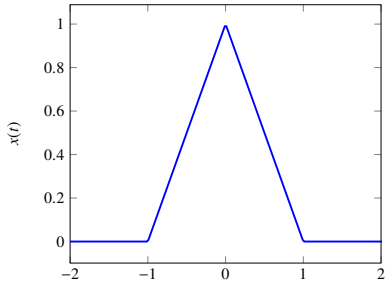
$$y(t) = \int_{-\infty}^{\infty} x_5(t-\lambda)x_5(\lambda)d\lambda = \int_0^t e^{-\alpha(t-\lambda)}e^{-\alpha\lambda}d\lambda = e^{-\alpha t}\int_0^t e^{-\alpha(\lambda-\lambda)}d\lambda = e^{-\alpha t}\int_0^t e^0 d\lambda$$

$$= e^{-\alpha t}\Big|_0^t \lambda = te^{-\alpha t}$$

$$y(t) = te^{-\alpha t} \cdot u_H(t)$$



**Task 2**




a) (3p.)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_0^1 (1-t)^2\, dt = 2\int_0^1 \left(t^2 - 2t + 1\right) dt$$

$$= 2\int_0^1 \left(\frac{t^3}{3} - t^2 + t\right) = \left(\frac{1}{3} - 1 + 1\right) = \frac{2}{3}$$

b) (2p.)

$$Y(f) = T\text{sinc}^2(fT)$$

c) (2p.)

$$Y(f) = \exp^{-j2\pi f t_0}\, \text{sinc}^2(f)$$

d) (3p.)

$$Y(f) = T\exp^{-j2\pi f t_0}\, \text{sinc}^2(Tf)$$

**Task 3**

a) (1p.)

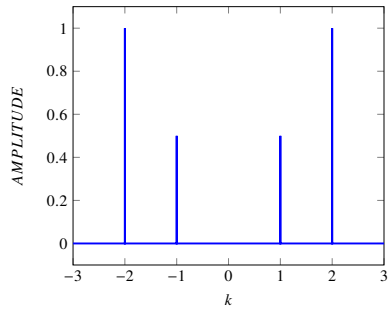
$$T_0 = \frac{1}{5}$$

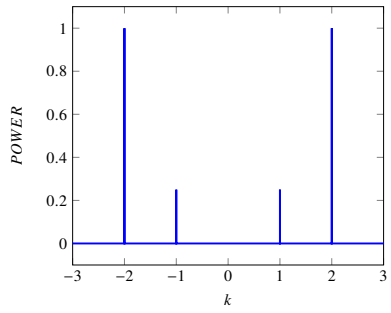
Signal contain frequencies 10 Hz and 5 Hz.

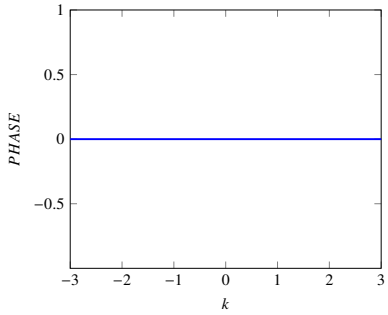
b) (3p.)

$$x_k = \begin{cases} 1, & |k| = 2 \\ 1/2, & |k| = 1 \\ 0, & \text{else} \end{cases}$$

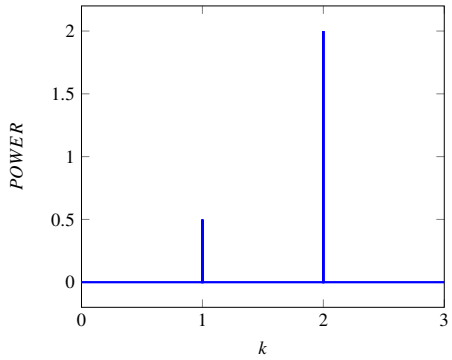
c) (2p.)




d) (2p.)




e) (2p.)

$$P = \frac{1}{2} + 2 = \frac{5}{2}$$