ELEC-E-5440 Statistical Signal Processing. Final Exam
December 12, 2019

1. Define or explain briefly the following concepts:

(a) Fisher Information

(b) Influence Function

(c) Bayes risk

(d) Divergence of Kalman Filter

(e) Consistency

(f) Array steering vector

Var

(g) Sufficient Statistic

(h) Signal and Noise Subspaces

min

2. Describe briefly the idea of Minimum Variance Distortionless Beamformer. What properties does this method have. How is its computation commonly done.

3. Let us have N independent and identically distributed observations x from the *pdf*

$$f_\theta(x) = e^{-(x-\theta)},$$

if $\theta < x < \infty$ and $f_\theta(x) = 0$ otherwise.

Find the Cramer-Rao lower bound for the variance of unbiased estimator of θ.

$-x + \theta = y$ ~~Lto~~ $\mathcal{L}(x|\theta) = \exp\left(N\theta - \sum_{i=1}^{N} x_i\right)$

$-dx = dy$

$\theta - x = a$ $L(x|\theta) = N\theta - \sum_{i=1}^{N} x_i$

$-dx = da$

$\mathcal{L} \quad \exp\left(N\theta - \sum_{i=1}^{N} x_i\right) -$

$$F_\theta(x \leq \theta) = \int_{-\infty}^{\theta} e^{-(x-\theta)}\, dx = \int_{-\infty}^{\theta} e^{a}\, da^{1} = -e^{-(x-\theta)} \Big|_{-\infty}^{\theta}$$

$$= -e^{-(\theta-\theta)} +$$

4.a Suppose we observe i.i.d measurements

$$y(k) = \alpha\ sin(\frac{k\pi}{2} + \phi) + v(k), \quad k = 1, ..., N$$

where $\alpha > 0$ and $\phi \in [-\pi, \pi]$ are deterministic and $v(k)$ is zero mean white Gaussian noise with variance σ^2 and N is even. Find Maximum Likelihood estimator of α and ϕ.

4.b Let us have N independent and identically observations $x_1, ..., x_N$ having the *pdf:*

$$f_\theta(x) = \theta^2 x e^{-\theta x},$$

where $\theta > 0$. Find the Maximum Likelihood estimator of θ.

5. Suppose that Θ is a random parameter and given $\Theta = \theta$, the observation y have a density

$$f(y|\theta) = (\theta/2)e^{-\theta|y|}, \ y \in R$$

$\int e^{-\theta|y|} d\theta$

Suppose further that Θ has prior density

$$f(\theta) = \begin{cases} 1/\theta, & 1 \le \theta \le e \\ 0 & \text{otherwise} \end{cases}$$

$-\theta|y| = a$

$-|y|\, d\theta = da$

$d\theta = -\frac{1}{|y|} da$

Find the MAP and Minimum Mean Square estimators of Θ based on observation.

Recall the Bayes rule:

$$f(\theta|\mathbf{y}) = \frac{f(\mathbf{y}|\theta)f(\theta)}{f(\mathbf{y})}$$

$\int -\frac{1}{|y|} e^{a} d\theta$

$= -\frac{1}{|y|} e^{-\theta|y|} \Big|_1^e$

e

$pq = \int p'q + \int pq'$