Prediction and Time Series Analysis — 2019
Exam
Answer to all the questions.

You are allowed to have pens and pencils, an eraser and a ruler, and one size A4 note (handwritten, text on one side only, name on the top right corner).

1. True or False (6 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, traditional least-squares estimators can be applied only if the residuals are normally distributed.

   (b) In the context of linear regression, the coefficient of determination is a measure of heteroscedasticity.

   (c) An autoregressive process of order 1 is always stationary.

   (d) The theoretical autocorrelation function of a pure autoregressive process of order 3 is equal to 0 after 3.

   (e) In exponential smoothing, the value of x_{t+1} is predicted using a weighted sum of the previous observation $x_t, x_{t-1}, x_{t-2}, \ldots$

   (f) Autoprojective time series models are models that involve only the time series to be forecasted.

2. Linear regression (6 p.)

   Consider a random sample $(x_1, y_1), (x_2, y_2), (x_3, y_3)\ldots, (x_{5117}, y_{5117})$ from the linear regression model

   $$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \epsilon_i.$$

   You have estimated the parameters β_0, β_1 and β_2 from the sample using traditional least squares estimators.

   (a) Explain, step by step, how to construct a 95% bootstrap confidence intervals for the parameters β_1 and β_2. (4 p.)

   (b) Assume that 0 is in the confidence interval that corresponds to β_1, but it is not in the confidence interval that corresponds to β_2. How would you interpret that? (2 p.)

3. Stationarity (6 p.)

Let x_t and z_t be weakly stationary stochastic processes such that, for all $t, s \in \mathbb{Z}$, we have that $E[x_t z_s] = 0$. Let $y_t = x_t + z_t$. Show that the process y_t is weakly stationary.

4. ARMA modeling (6 p.)

Assume that you have observed a series $x_0, x_1, x_2, \ldots, x_{7305}$.

(a) Based on plotting the series, you observe a linear trend. You manage to stationarize the process by taking a difference. Give the elements of the obtained stationary process in terms of the elements of the original observed series. (1 p.)

(b) Based on plotting the stationarized series and its estimated autocorrelation and partial autocorrelation -functions, you think that the stationarized series is a pure autoregressive process of order 2. Give the definition of an autoregressive process of order 2. (2 p.)

(c) You decide to apply traditional ARMA-modeling based prediction to calculate the 1, 2, 3 and 4 step predictions for the stationarized series. What are the predicted values of x_{7306}, x_{7307}, x_{7308} and x_{7309} of the original observed series? (3 p.)

5. Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

(a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(2)-process?

(b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?

(c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(1)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SAR(2)_6$-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SAR(2)_3$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

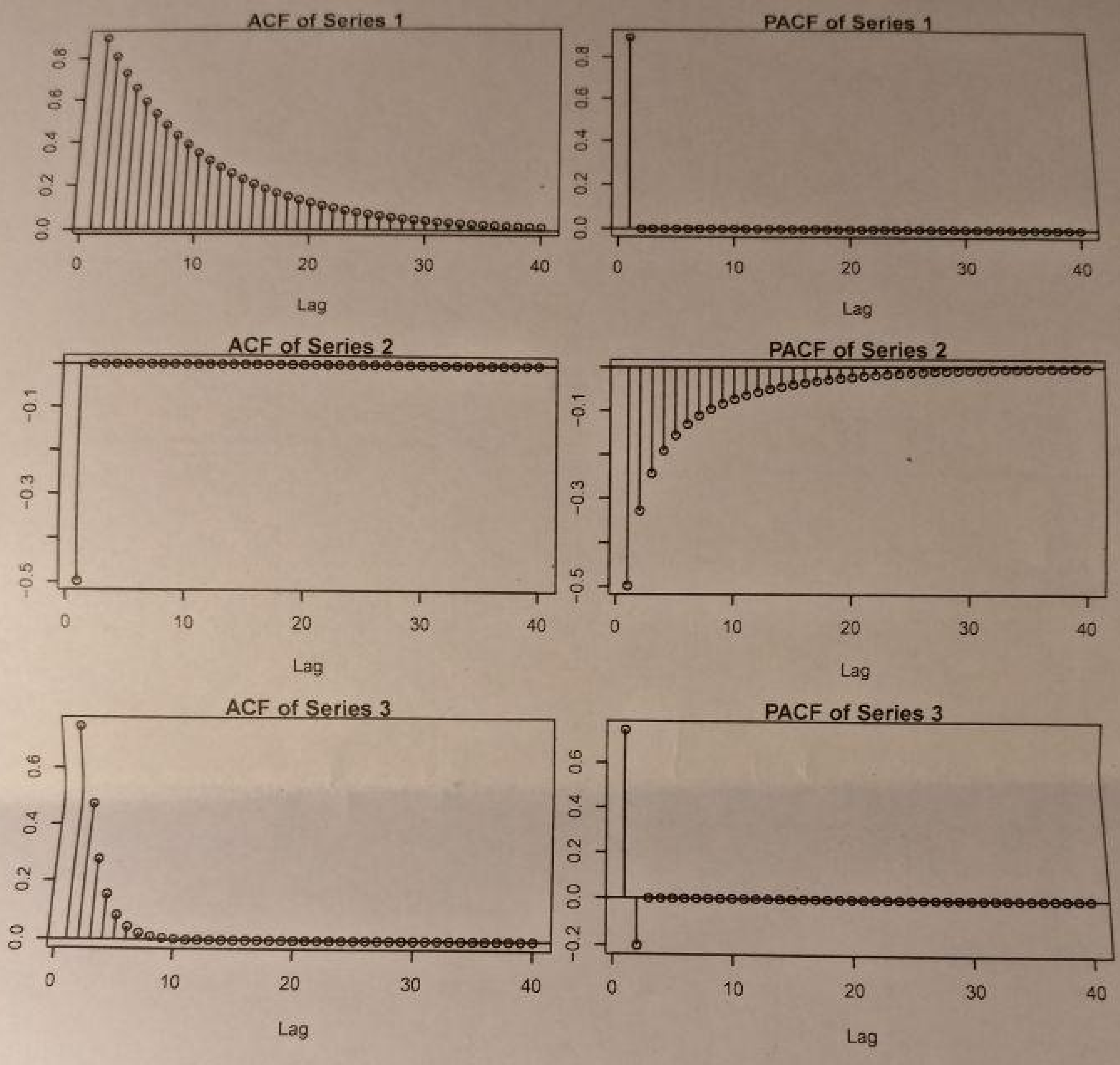
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag
0.0
0.2
0.4
0.6
0.8
-0.1
-0.3
-0.5
-0.2
0
10
20
30
40

Figure 1

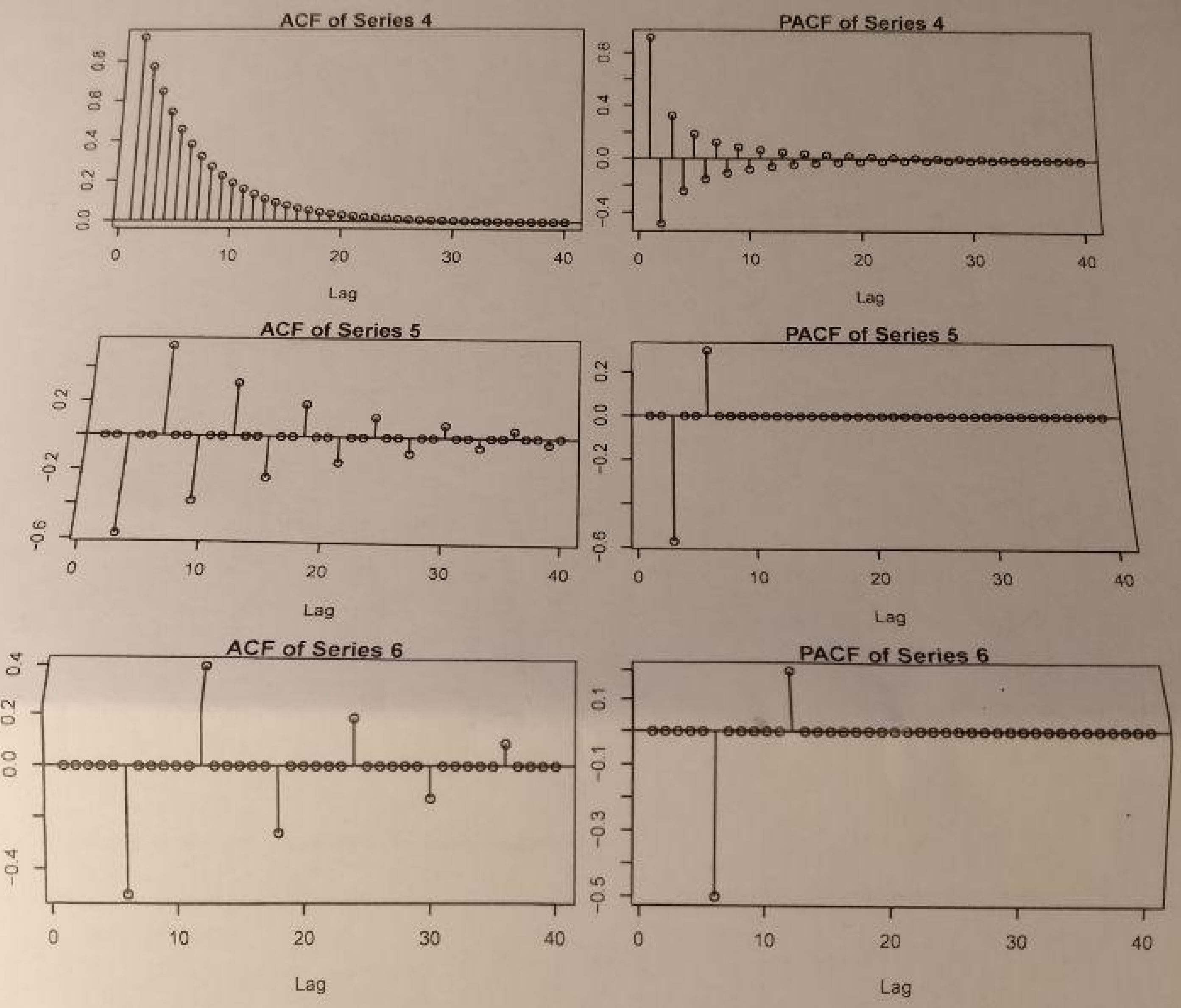
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
Lag

Figure 2