Instructions. You must answer *all questions* in order to pass the exam. If you cannot solve a problem entirely, try to at least explain what you tried and what went wrong; just make sure that you do not leave any question unanswered. In all questions, *you can use any results from the textbook*.

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Definitions. Let $G = (V, E)$ be a graph. We say that $X \subseteq V$ is a *clique* if for all nodes $a, b \in X$, $a \neq b$, there is an edge $\{a, b\} \in E$. The *size* of the clique is the number of nodes in set X.

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Question 1: Covering maps. Prove that the following problem cannot be solved with any deterministic algorithm in the PN model:

- Graph family: connected graphs with exactly 10 nodes.
- Local inputs: nothing.
- Local outputs: all nodes have to output the size of the largest clique in the input graph.

(For example, if the input graph contains a clique with 4 nodes and it does not contain a clique with 5 nodes or more, then all nodes must output the same value, 4.)

Question 2: Local neighbourhoods. Prove that the problem from question 1 cannot be solved in 5 communication rounds in the LOCAL model with any deterministic distributed algorithm.

Question 3: Graph theory & randomness. It is enough to answer either part (a) or (b):

(a) Let $G = (V, E)$ be any *regular* graph with 5 nodes (that is, all nodes have the same degree). Prove: the size of the largest clique in G cannot be 3 (that is, the largest clique has to be smaller than 3 nodes or larger than 3 nodes, but it cannot be exactly 3 nodes).

(b) Design a *randomized* distributed algorithm that solves the problem from question 1 in the PN model. Any running time is fine. The algorithm has to be a *Las Vegas algorithm*, i.e., when it terminates, it always produces a correct output. It is sufficient to give a *brief, informal description* of the algorithm—you do not need to use the precise state-machine formalism, and you do not need to prove that your algorithm is correct.