Aalto University School of Science, Department of Computer Science
JR & TJ

CS-E3220 Declarative Programming

Examination, December 16th, 2019

Use of electronic devices, such as calculators, is not allowed in the examination.

Assignment 1 Formulas and normal forms. (Max. 5p)

Consider the propositional formula $(a \oplus b) \vee c$.

1. Derive a logically equivalent formula that is in the conjunctive normal form.
2. Derive a logically equivalent formula that is in the disjunctive normal form.

Assignment 2 DPLL and Resolution (Max. 5p)

Consider the unsatisfiable CNF formula $(\neg a \vee b) \wedge (a \vee b) \wedge (\neg b \vee c \vee d) \wedge (\neg b \vee \neg d) \wedge (\neg c \vee d)$. Give a DPLL search tree and a resolution proof of unsatisfiability for it.

Assignment 3 CSPs and Arc Consistency (Max. 5p)

Consider the all-different constraint $\text{ALLDIFFERENT}(x_1, x_2, x_3, x_4)$ when the domains are $D(x_1) = \{2,4\}$, $D(x_2) = \{1,2,3\}$, $D(x_3) = \{2,3,4,5\}$, and $D(x_4) = \{2,4\}$.

Does the constraint have solutions? If it has, give one. If it is not, argue why this is the case.

Is the constraint arc-consistent? If it is, explain why this is the case. If it is not, propagate the constraint to be arc-consistent and give the resulting new domains.

Assignment 4 Binary Decision Diagrams (Max. 5p) Consider variables A, B, C, D, in this variable ordering. Construct an OBDD (ordered reduced binary decision diagram) for the Boolean function that is true if and only if no two consecutive variables are *true*. This can be done by forming the OBDD for the conjunction of $\neg(A \wedge B)$, $\neg(B \wedge C)$, and $\neg(C \wedge D)$, by using the Apply function, but it is far easier to form it directly.

State the conditions for OBDDs being reduced and ordered, and argue that your OBDD satisfies them.

Assignment 5 Modal logics (Max. 5p)

For each of the following formulas, give a model in which the formula is true.

1. $(\Diamond\Box a) \wedge (\Diamond\Box\neg a) \wedge (\Diamond\Diamond a) \wedge (\Diamond\Diamond\neg a) \wedge (\Box a) \wedge \neg a$
2. $\mathcal{G}((\mathcal{F} a) \wedge (\mathcal{F} \neg a))$

The second formula is for the Linear Temporal Logic LTL.

Assignment 6 Weakest preconditions (Max. 5p)

For the following programs π and formulas ϕ, find the weakest precondition, that is, the weakest formula ψ the satisfaction of which guarantees that after executing π, the formula ϕ holds.

π	ϕ
$x := y + 10$	$x < z + 5$
$x := x + 1$; IF $x < y$ THEN $z := x$ ELSE $z := y$	$z < 10$

NB: Assignments continue on the other side of the question sheet!

Assignment 7 State-space search with propositional logic (Max. 5p)

Consider the following reachability problem in the propositional logic.

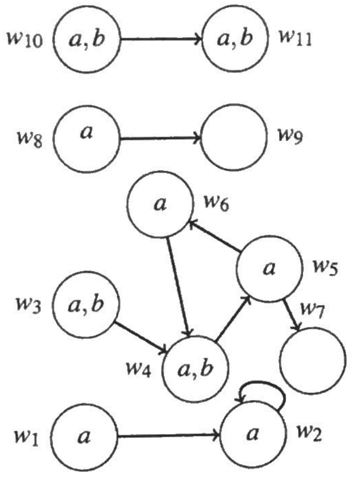
The states are encoded by the state variables a, b and c. The initial state is 001 (meaning that a and b are *false* and c is *true*.) The goal states are those in which the formula $a \wedge \neg b$ is *true*. The actions are

- inverting the values of all state variables (e.g. turning 010 to 101), and
- rotating the values of a, b and c one step to the left (e.g. turning 001 to 010, and 100 to 001).

Questions:

1. Give a propositional formula that represents transition sequences that start from the initial state, end in a goal state, and have length 2 (= two consecutive actions, three consecutive states.)
2. How many satisfying assignments does your formula have?
3. Give a satisfying assignment (values of all propositional variables), or argue why the formula is not satisfiable.

Assignment 8 (Max. 5p)



Run the CTL model-checking algorithm for the formula $\phi = b \wedge E\mathcal{G}a$ and the model on the left. For all of the relevant subformulas of ϕ, list which worlds will be labelled with that subformula.

Assignment 9 Satisfiability Modulo Theories (Max. 10p)

(a) Recall that T_{EUF} is the theory of uninterpreted functions and predicates (with built-in equality, as usual). Which of the following are true? Justify your answer by giving a model or a proof.

1. $a \approx c \wedge a \not\approx b \wedge g(f(a),b) \approx g(f(c),a)$ is T_{EUF}-satisfiable.
2. $p(x) \wedge f(f(x)) \approx x \wedge f(f(f(f(f(x))))) \approx x \wedge \neg p(f(x))$ is T_{EUF}-unsatisfiable.

(5p)

(b) Use the method presented in the lecture slides (constraint graphs and the Bellman–Ford algorithm) to check whether the following conjunction

$$(x-y \leq 1) \wedge (x-z \leq 3) \wedge (y-z \leq -2) \wedge (z-w \leq -1) \wedge (w-x \leq 4)$$

is *IDL*-satisfiable, where *IDL* is the integer difference logic. If it is not, find a negative cycle and give the corresponding *IDL*-unsatisfiable sub-conjunction. If it is, give a model for the conjunction.

(5p)

Assignment 10 When did you complete your answers (please record the time)?