

## ELEC-E8104 Stochastics models and estimation, 5 ST

Exam 22.10.2019

It is allowed to use the delivered Collection of important formulas for this course.

1. Consider one measurement  $z$  of an unknown constant parameter  $x$ .

$$z = 3x + w$$

Let's assume that  $w$  has Gaussian pdf with mean zero and covariance  $P_{zz}$ .

Find ML-estimator for the parameter  $x$ . How could you characterize MAP and ML estimators and the relations between them?

(6 p)

2. Find a discrete extended Kalman filter for a tractor having the following continuous time model, in which 2D-position, heading angle  $\theta$  and steering angle  $\alpha_F$  are state variables

$$\dot{x}_R = v_R \cos(\theta)$$

$$\dot{y}_R = v_R \sin(\theta)$$

$$\dot{\theta} = \frac{1}{a} v_R \tan(\alpha_F)$$

$$\dot{\alpha}_F = \omega_F$$

As inputs,  $v_R$  is the velocity,  $\omega_F$  is the angular velocity of the steering angle;  $a$  is a constant. The position is measured with a good GPS disturbed with Gaussian white noise. The heading angle is measured with a quite inaccurate magnetometer, which has Gaussian white measurement error.

The system can be discretized with Euler method, which can be reasoned with the definition for derivative.

$$\dot{x} = f(x, u, t) \approx \frac{x(k+1) - x(k)}{T}$$

(6 p)

3. Explain the meaning of the Principle of orthogonality in connection of the linear MMSE-estimation.

(6 p)

4. a) In what cases it is beneficial to use Information filter? in what cases 'normal' Kalman filter?

(2 p)

- b) Why a posteriori covariance does not converge into zero in Kalman filter?

(2 p)

- c) What is the meaning of the residual covariance?

(2 p)

5. How a priori estimate can be calculated in the case of nonlinear estimation?

(6 p)