

ELEC-E8104 Stochastics models and estimation, 5 SP

Exam 18.12.2019

It is allowed to use the delivered Collection of important formulas for this course.

1. Consider one measurement z of an unknown constant parameter x .

$$z = 4x + w$$

Let's assume that w has Gaussian pdf with mean zero and covariance P_{zz} .

Find the ML-estimator for the parameter x . Find also the LS estimator for parameter x and characterize the relationship between ML and LS.

(6 p)

2. A following continuous time model represents a robot moving in 2D. The position (x, y) , heading angle θ and velocity v are state variables

$$\dot{x}_R = v_R \cos(\theta)$$

$$\dot{y}_R = v_R \sin(\theta)$$

$$\dot{\theta} = \frac{1}{a} v_R \tan(u_1)$$

$$\dot{v}_R = u_2$$

As inputs, u_1 is the steering angle, u_2 is the acceleration. a is a constant.

As the measurements, two radars having fixed (x, y) -positions $(0, 0)$ and $(2, 0)$ give the Euclidian distances z_1 and z_2 to the vehicle.

a) Write the continuous nonlinear state space equations and discretize the continuous time system using Euler method.

b) Calculate the Jacobians that are needed for implementing an EKF.

The system can be discretized with Euler method, which can be reasoned with the definition for derivative.

$$\dot{x} = f(x, u, t) \approx \frac{x(k+1) - x(k)}{T}$$

(6 p)

3. What is the optimal nonlinear state estimator? Exact equations are not required. Why, in cases of nonlinear systems, the implemented state estimators are almost always only approximations of the optimal state estimators?

(6 p)

4. a) In what cases it is beneficial to use Information filter? In what cases 'normal' Kalman filter?

b) What is information state?

c) When the constant gain Kalman filter can be used?

(6 p)

5. More questions about the normal (linear) Kalman filter:

a) Describe what are the presumptions made about the dynamic model, measurements and the associated noises.

b) What is the meaning of the residual covariance?

d) Why a posteriori covariance is always 'smaller' than a priori covariance in Kalman filter?

(6 p)