MS-E2121 – Linear Optimization 12.12.2019
Final exam R. Lahdelma

Max exam points is 25. These will be scaled linearly into the range [0,65] and added to assignment points.

1. Are the following statements true (T) or false (F). Correct answer = +0.5, incorrect = -0.5 points. Total number of points is truncated into range [0,7].

Consider an LP problem in standard (canonical) form with m constraints and n variables:

a) A basic feasible solution (BFS) can have more than m positive components.
b) A BFS can have fewer than m positive components.
c) A basic solution (BS) can have negative components.
d) A BS can have more than n-m zero components.
e) A BS can have fewer than n-m zero components.
f) Let f **x** and **p** be solutions to the LP and its dual problem, determined by the same basis B. Then **x** and **p** satisfy the complementary slackness conditions.
g) If **x** and **p** (above) satisfy the complementary slackness conditions, then they are optimal solutions.
h) If the LP problem is unbounded, its dual problem must be infeasible.
i) If the LP problem is infeasible, its dual problem must be unbounded.
j) If the dual of the LP problem is infeasible, then the primal problem can be infeasible.
k) If the LP problem has an optimal solution, its dual problem is feasible.
l) The reduced costs of a basic variable can be non-zero.
m) LP applies only for linear problems.
n) Since a polyhedron has a finite number of extreme points, any LP problem has a finite number of optimal solutions.

2. Solve the LP problem by tabular simplex using the Two-Phase Simplex method. (6p)

$$\text{(P) Minimize } 2x_1$$
$$\text{s.t.}$$
$$x_1 - x_3 = 3$$
$$x_1 - x_2 - 2x_5 = 1$$
$$2x_1 + x_4 \leq 7$$
$$x_1, x_2, x_3 \geq 0$$

3. Consider the following LP:

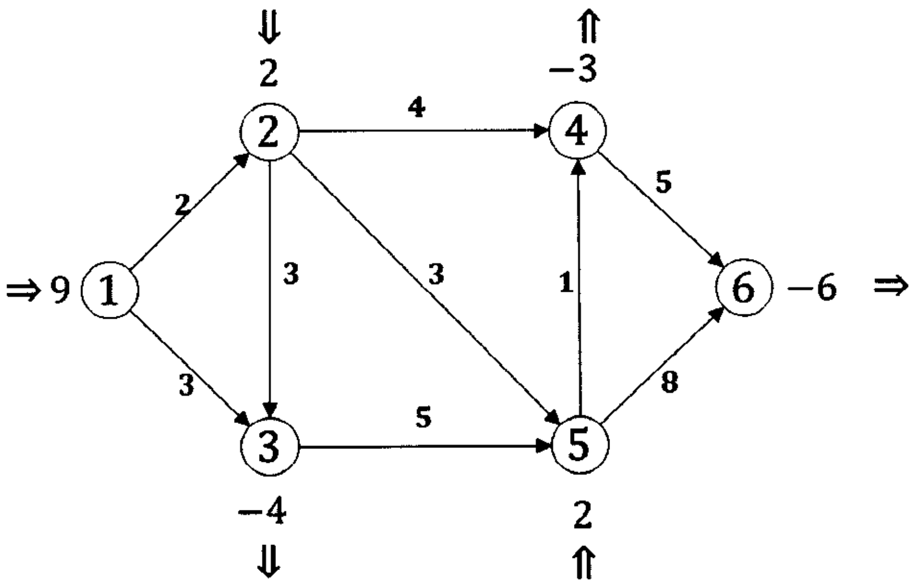
$$\text{(P) Minimize } -x_1 - 2x_2 + 4x_3 + 5x_5$$
$$\text{s.t.}$$
$$x_1 + x_3 - 2x_4 - x_5 + 2x_6 = 3$$
$$x_1 + x_2 - x_4 - 3x_5 + 3x_6 + x_7 = 7$$
$$-x_1 - 2x_2 - x_3 - x_4 + 5x_5 - 2x_6 \leq 4$$
$$x_1, \ldots, x_7 \geq 0\,.$$

a) Write the dual of P.(2pt)

b) State the complementary slackness theorem.(2pt)

c) Use the complementary slackness theorem to verify if the solution $x_1 = 3$, $x_2 = 4$, $x_j = 0, j = 3, \ldots, 7$ is optimal for P and justify your answer.(2pt)

4. Consider the uncapacitated minimum cost flow problem defined by the graph below. The number on each arc indicates its cost. Node supplies are indicated by the double arrows.



Solve the problem by using the Network Simplex algorithm starting from the tree solution defined by the arcs (1, 2), (2, 3), (2, 4), (5, 4), (4, 6). (6pt)

At each iteration:

- Indicate the set T and report the corresponding tree solution.
- Report the dual variables at each iteration.
- Explain how the leaving and entering variables are selected.
- Explain how the flows are updated.