ELEC-E8101 Digital and Optimal Control

Final Exam (11.12.2019)

- *Write your name and student number to each answer sheet.*
- *All four (4) problems must be answered.*
- *No other literature except the "Digital and Control Databook" is allowed. The databook must be returned, if you have received it from an invigilator.*
- *Electronic calculators can be used. No calculators with capabilities for storing and displaying verbal information are allowed.*

Good luck!

B

(Turn over)

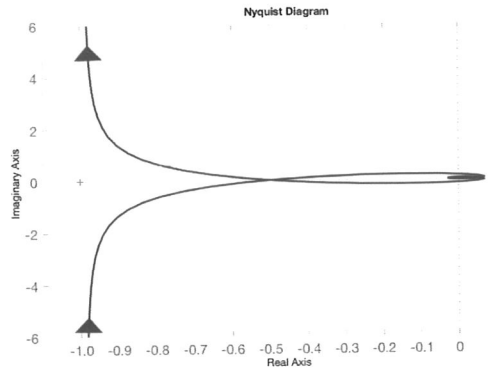
1. Solve the following problems:

a) Determine whether the following equation

$$z^3 - 2z^2 + 2z - 0.5 = 0$$

has all its roots inside the unit disc. [2p]

b) The Nyquist plot for the plant $H(z) = \frac{0.1054}{z^3 - 1.7z^2 + 0.8z - 0.1}$ is:



Determine approximately (from the Nyquist diagram) the values of the gain K for which the closed loop system is stable. [1p]

c) Find the z-transform of the sequence obtained by sampling, (with uniform sampling period h), the continuous-time waveform whose Laplace transforms is [2p]

$$\frac{1}{s^2 + 4s + 3}$$

d) A stochastic process is generated as

$$x[k+1] = 0.5x[k] + v[k]$$
$$y[k] = x[k] + e[k]$$

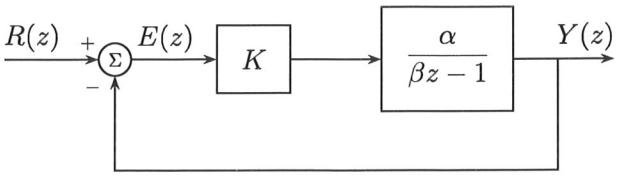
where v and e are uncorrelated white-noise processes with covariances r_1 and r_2, respectively. Further, $x[0]$ is normally distributed with zero mean and standard deviation σ.

(i) Determine the Kalman filter for the system. [2p]

(ii) What is the Kalman gain K in steady state? [2p]

(iii) How would the Kalman gain change if the standard deviation σ of $x[0]$ change? [1p]

2. The block diagram of a discrete-time feedback control system is shown below, where α and β are constants satisfying $\alpha > 0$ and $\beta > 0$.



a) Determine the range of values for K, such that the closed-loop system is stable as a function of α and β. [3p]

b) For the range of values of K found in part a), find the steady-state of the value of the error $e[k]$ (i.e., $\lim_{k\to\infty} e[k]$) when the input $r[k]$ is a unit step. [3p]

c) With $\alpha = 1$, for what values of β and K is the steady-state error $e[k]$ equal to zero? [2p]

d) For $\alpha = 1$, $\beta = 2$, what values of K result in a steady-state error of less than 1% ? [2p]

3. Suppose a discrete time system is governed by the difference equation:

$$y[k+1] + by[k] = u[k] - au[k-1]$$

with $u[-1] = 0$ and $y[0] = \phi$ (ϕ is a constant).

a) Using the z-transform, compute $y[k]$ for $u[k] = \{1\}_{k\geq 0}$ (step input). [3p]

b) Let $\phi = 0$.

i) Compute the transfer function of the system. [3p]

ii) Under what conditions will the system be stable? [2p]

c) Keeping $\phi = 0$, introduce feedback of the form $u[k] = -\ell y[k] + x[k]$, where $x[k]$ is the new input to the system.

i) Compute the transfer function of the new system. [3p]

ii) For the case $a = b$, provide conditions for stability in terms of a and ℓ. [3p]

iii) Is the system causal? [1p]

(Turn over)

4. Consider the following simplified model of DC-motor

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s(s+1)}$$

a) Assuming ZOH and sampling time h, show that the the corresponding discrete time state-space representation is given by [5p]

$$\begin{cases} \mathbf{x}(kh+h) = \begin{bmatrix} e^{-h} & 0 \\ 1-e^{-h} & 1 \end{bmatrix} \mathbf{x}(kh) + \begin{bmatrix} 1-e^{-h} \\ h-1+e^{-h} \end{bmatrix} u(kh) \\ y(kh) = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x}(kh). \end{cases}$$

b) Let $h = 1$. Design for the discretized system a state feedback control law of the form

$$u[k] = ar - L\mathbf{x}[k],$$

where a is constant r is the reference and L a gain vector. Assume that the states are measurable, the desired closed loop poles are $0.5 \pm j0.2$ and the static gain of the closed loop system is 1. [5p]

c) Suppose now that only the input u and output y are measured (i.e., x_1 is not measured). Design a discrete-time controller to the process, which consists of a combination of state feedback and state observer as

$$u[k] = ar - L\hat{\mathbf{x}}[k].$$

The desired closed loop poles due to state feedback are, as before, $0.5 \pm j0.2$ and the observer has the dead-beat characteristics. Choose the pre-compensator coefficient a such that the output of the controlled process follows the reference. [5p]