AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo / Markus Mattila

Exam, 3.4.2018

A function (non-graphing) calculator is allowed.

1. Use tabular Simplex algorithm in the following problem:

$$\begin{array}{lrcl} \max & x_1 & & \\ \text{s.t.} & 2x_1 - x_2 & \leq & 2 \\ & -x_1 + x_2 & \leq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

2. Briefly define the following terms:

   a) Slack variable (1p)
   b) Nash equilibrium (1p)
   c) Pareto optimal solution (1p)
   d) Binary variable (1p)
   e) Shadow price (1p)
   f) Convex set (1p)

3. Consider a problem

$$\begin{array}{lrcl} \min & (x_1-3)^2 + (x_2-3)^2 & & \\ \text{s.t.} & x_1^2 + x_2 & \leq & 4 \\ & -x_1 + x_2 & = & 2 \\ & x_2 & \geq & 0, \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Show that the necessary Karush-Kuhn-Tucker (KKT) conditions are satisfied in the optimal solution point. (4p)

4. The mighty Hun emperor Mukbar Attila is planning a conquest in Europe. He has narrowed his options down to four vulnerable cities: Paris, London, Rome and Constantinople. M. Attila has an army of 10 000 Huns at his disposal.

   The potential loot of each target (millions of denars) and the troops required for conquest are presented in Table 1.

   Table 1: Target information

	Loot	Required troops
Paris	2 M	2500
London	1 M	1500
Rome	5 M	5000
Constantinople	4 M	3500

   M. Attila does not want to spread his troops too much - therefore, he cannot attack both Rome and London. In addition, the tactically gifted emperor decides that he cannot conquer London without also overtaking Paris.

   a) Formulate the problem as a linear integer optimization problem, when M. Attila wants to maximize the total amount of received loot. (4p)
   b) After some thinking, M. Attila concludes that he wants a minimum loot of 10 million denars. Formulate the resulting goal programming problem, when the loss of 1 million denars is equally acceptable to hiring 500 more soldiers. (2p)
   Neither case needs to be solved.

5. Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

   a) For a linear programming (LP) problem, there is always either exactly one solution or no solution at all.
   b) It is possible to formulate such an optimization problem, which has exactly two solutions.
   c) Consider a linear programming (LP) problem. If the solution of the primal problem is unbounded, then the dual problem has no feasible solution.

AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo / Markus Mattila

Exam, 25.5.2018

A function (non-graphing) calculator is allowed.

1. Use tabular Simplex algorithm in the following problem:

$$\begin{array}{rrcl} \max & 2x_1 + 3x_2 & & \\ \text{s.t.} & 2x_1 + 3x_2 & \leq & 6 \\ & x_1 - x_2 & \leq & 1 \\ & x_2 & \leq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

2. Briefly define the following terms:

   a) Slack variable (1p)
   b) Nash equilibrium (1p)
   c) Pareto optimal solution (1p)
   d) Binary variable (1p)
   e) Newton's method (1p)
   f) Convex set (1p)

3. Consider a problem

$$\begin{array}{lll} \max\ (x_1-3)^2 + x_2^2 & & \\ \text{s.t.}\quad x_1 & \geq & 1 \\ \qquad x_1^2 - 2 & \leq & x_2 \\ \qquad 2x_1 - 2 & = & x_2\ . \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Show that the necessary Karush-Kuhn-Tucker (KKT) conditions are satisfied in the optimal solution point. (4p)

4. Vesa is a friend of good beer. He is on a beer journey in Belgium and planning exports to home in a beer store. For financial and transportation reasons, he has decided to buy at most three different beer types: his alternatives are blonde, dubbel, and lambic beer types. Each beer type is available in 0.33 and 0.75 liter bottles. The price list of the bottles is presented in Table 1.

Table 1: Price list

	0.33 l	0.75 l
Blonde	€1.5	€3.5
Dubbel	€2	€3.5
Lambic	€4.5	€6

Small bottles are packed into small crates, and big bottles are packed into big crates. Small bottles cannot be packed into big crates or vice versa. One small crate has a space for 24 small bottles and one big crate for 10 big bottles. Vesa is planning his shoppings so that at least half of the bottles are of lambic type, and the number of blonde type bottles must be greater than or equal to the number of dubbel type bottles. Vesa has €150 to spend and he wishes to maximize the amount of bought beer measured in litres. Formulate the problem as a linear integer programming problem when

a) Vesa's car has a space for arbitrarily many crates and crates are free. (3p)
b) Vesa's car has a space for two small crates and one big crate. A small crate costs €5 and a big crate €6. (3p)
Neither case needs to be solved.

5. Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

a) For an optimization problem, there is always either exactly one solution or no solution at all.
b) It is possible to formulate an optimization problem that has exactly two solutions.
c) The solution of an optimization problem is always located in a corner point of the feasible region.

AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo / Markus Mattila

Exam, 12/2018

A function (non-graphing) calculator is allowed.

1. Use tabular Simplex algorithm in the following problem:

$$\begin{array}{lrcl} \max & x_1 & & \\ \text{s.t.} & 2x_1 - x_2 & \leq & 2 \\ & -x_1 + x_2 & \leq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

2. Briefly define the following terms:

   a) Simplex iteration (1p)
   b) Nash equilibrium (1p)
   c) Portfolio optimization (1p)
   d) Subtour (1p)
   e) Dual variable (1p)
   f) Convex function (1p)

3. Consider a problem

$$\begin{array}{lrcl} \min & (x_1-3)^2 + (x_2-3)^2 & & \\ \text{s.t.} & x_1^2 + x_2 & \leq & 4 \\ & -x_1 + x_2 & = & 2 \\ & x_2 & \geq & 0, \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Show that the necessary Karush-Kuhn-Tucker (KKT) conditions are satisfied in the optimal solution point. (4p)

4. Consider a problem

$$\begin{array}{rlcl} \max & 9x_1 + 5x_2 & & \\ \text{s.t.} & 4x_1 + 9x_2 & \leq & 35 \\ & x_1 & \leq & 6 \\ & x_1 - 3x_2 & \geq & 1 \\ & 3x_1 + 2x_2 & \leq & 2 \\ & x_1, x_2 & \in & \mathbb{Z}_+ \cup \{0\} \end{array}$$

a) Solve the LP relaxation of the problem graphically. (1p)
b) Determine the solution to the problem with Branch & Bound algorithm. Solve subproblems graphically. (3p)
c) Draw the course of your solution in tree form, and justify with it that the solution you got is the best possible integer solution to the original problem. (2p)

5. Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

a) For a linear programming (LP) problem, there is always either exactly one solution or no solution at all.
b) It is possible to formulate such an optimization problem, which has exactly two solutions.
c) Consider a linear programming (LP) problem. If the solution of the primal problem is unbounded, then the dual problem has no feasible solution.