AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo / Markus Mattila

Exam, 4.4.2017

A function (non-graphing) calculator is allowed.

1. Use tabular Simplex algorithm in the following problem:

$$\begin{array}{lrcl} \max & 2x_1 + x_2 & & \\ \text{s.e.} & x_1 & \leq & 2 \\ & x_1 + 2x_2 & \leq & 6 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

2. Briefly define the following terms:
   - slack variable
   - binary choice
   - subtour
   - function f is convex
   - portfolio optimization
   - shadow price

3. Consider a problem

$$\begin{array}{lrcl} \min & (x_1 - 8)^2 + (x_2 - 6)^2 & & \\ \text{s.e.} & x_1^2 + x_2^2 & \leq & 25 \\ & x_1 + 3x_2 & \leq & 15 \\ & x_1, x_2 & \geq & 0, \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Show that the necessary Karush-Kuhn-Tucker (KKT) conditions are satisfied in the optimal solution point. (3p)
   c) What is the solution if the objective function is $f(x_1, x_2) = (x_1 - 3)^2 + (x_2 - 1)^2$ and the feasible region remains the same?

4. The mighty Hun emperor Mukbar Attila is planning a conquest in Europe. He has narrowed his options down to four vulnerable cities: Paris, London, Rome and Constantinople. M. Attila has an army of 10 000 Huns at his disposal.

   The potential loot of each target (millions of denars) and the troops required for conquest are presented in table 1.

   Table 1: Target information

	Loot	Required troops
Paris	2 M	2500
London	1 M	1500
Rome	5 M	5000
Constantinople	4 M	3500

   M. Attila does not want to spread his troops too much - therefore, he cannot attack both Rome and London. In addition, the tactically gifted emperor decides that he cannot conquer London without also overtaking Paris.

   a) Formulate the problem as a linear integer optimization problem, when M. Attila wants to maximize the total amount of received loot. (4p)
   b) After some thinking, M. Attila concludes that he wants a minimum loot of 10 million denars. Formulate the resulting goal programming problem, when the loss of 1 million denars is equally acceptable to hiring 500 more soldiers. (2p)
   Neither case needs to be solved.

5. a) Derive a 1-dimensional secant method starting from Newton's method. (2p)
   b) Describe the steps of an n-dimensional gradient method. (4p)

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A non-graphical calculator is allowed

1. Consider a problem

$$\begin{array}{lll} \max & 4x_1 + 3x_2 & \\ \text{s.e.} & x_1 + x_2 & \leq 3 \\ & -3x_1 + 5 & \geq x_2 \\ & 2x_1 - x_2 & \geq -1 \\ & x_1, x_2 & \geq 0. \end{array}$$

   a) Transform the linear problem into the standard form (1p)
   b) Solve the LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show the progress of the Simplex algorithm in the region. (2p)

2. Briefly define the following terms:

   a) Slack variable (1p)
   b) Nash equilibrium (1p)
   c) Pareto optimal solution (1p)
   d) Binary variable (1p)
   e) Shadow price (1p)
   f) Convex set (1p)

3. Consider a problem

$$\begin{array}{lrll} \min & (x_1 - 5)^2 + (x_2 - 5)^2 & & \\ \text{s.e.} & x_1^2 + x_2^2 & \leq & 25 \\ & x_1 & \leq & 3 \\ & x_1, x_2 & \geq & 0, \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and the contour lines of the objective function. (2p)
   b) Show that the necessary Karush-Kuhn-Tucker (KKT) conditions are satisfied in the optimal solution point. (4p)

4. Consider a problem

$$
\begin{array}{lrcl}
\max & 3x_1 + 2x_2 & & \\
\text{s.e.} & x_1 + 2x_2 & \leq & 3 \\
 & 2x_1 + 2x_2 & \leq & 5 \\
 & x_1, x_2 & \in & \mathbb{Z}_+
\end{array}
$$

a) Solve the LP relaxation of the problem graphically. (2p)
b) Solve the original problem by using the Branch-and-Bound method. Solve the subproblems graphically. (3p)
c) Present your solution in a tree form. With it explain why the solution you found is indeed the optimal integer solution. (1p)

5. a) Derive a 1-dimensional secant method starting from Newton's method. (2p)
b) Describe the steps of an n-dimensional gradient method. (4p)