AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo / Vesa Husgafvel

Exam, 5.4.2016

A function calculator is allowed. A graphing calculator or any other tools are not permitted.

1. Briefly define the following terms of the Simplex method:
   a) pivot element (1p)
   b) exiting variable (1p)
   c) optimality condition (2p)
   d) feasibility condition (2p)

2. Use tabular Simplex algorithm in the following problem:

$$\begin{array}{rrcl} \max & 2x_1 + 3x_2 & & \\ \text{s.t.} & x_1 + 3x_2 & \leq & 5 \\ & 2x_1 - 2x_2 & \leq & 3 \\ & x_1 + 2x_2 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the Simplex method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

3. Consider a problem

$$\begin{array}{llcl} \max & x_1^2 + (x_2 - 4)^2 & & \\ \text{s.t.} & x_1 + x_2 & = & 2 \\ & x_1^2 + x_2 & \leq & 4 \\ & x_1 & \leq & 1 \\ & x_2 & \geq & 0 \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Present the necessary Karush-Kuhn-Tucker (KKT) conditions and examine if the solution satisfies these conditions. (4p)

4. Vesa is a friend of good beer. He is on a beer journey in Belgium and planning exports to home in a beer store. For financial and transportation reasons, he has decided to buy at most three different beer types: his alternatives are blonde, dubbel, and lambic beer types. Each beer type is available in 0.33 and 0.75 liter bottles. The price list of the bottles is presented in Table 1.

Table 1: Price list

	0.33 l	0.75 l
Blonde	€1.5	€3.5
Dubbel	€2	€3.5
Lambic	€4.5	€6

Small bottles are packed into small crates, and big bottles are packed into big crates. Small bottles cannot be packed into big crates or vice versa. One small crate has a space for 24 small bottles and one big crate for 10 big bottles. Vesa is planning his shoppings so that at least half of the bottles are of lambic type, and the number of blonde type bottles must be greater than or equal to the number of dubbel type bottles. Vesa has €150 to spend and he wishes to maximize the amount of bought beer measured in litres. Formulate the problem as a linear integer programming problem when

a) Vesa's car has a space for arbitrarily many crates and crates are free. (3p)
b) Vesa's car has a space for two small crates and one big crate. A small crate costs €5 and a big crate €6. (3p)
Neither case needs to be solved.

5. a) Derive a 1-dimensional secant method starting from Newton's method. (2p)
b) Describe the steps of an n-dimensional gradient method. (4p)

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MS-C2105 Introduction to optimization Harri Ehtamo / Markus Mattila

Exam, December 19th 2016

- Briefly define the following terms.

  a) Surplus variable (1p)
  b) Shadow price (1p)
  c) Binary variable (1p)
  d) Function f is convex (1p)
  e) Efficient, or Pareto optimal solution (1p)
  f) Portfolio optimization (1p)

- Use tabular Simplex algorithm in the following problem:

$$\begin{array}{rrcl} \max & 2x_1 + 3x_2 & & \\ \text{s.t.} & x_1 + 3x_2 & \leq & 5 \\ & 2x_1 - 2x_2 & \leq & 3 \\ & x_1 + 2x_2 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$

  a) Transform the linear problem into the standard form. (1p)
  b) Solve LP problem using the Simplex method. (3p)
  c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

- Consider a problem

$$\begin{array}{lllr} \min & (x_1 - 5)^2 + (x_2 - 4)^2 & & \\ \text{s.e.} & x_1^2 - 4x_1 - x_2 + 5 & \leq & 0 \\ & 2x_1 + 3x_2 - 12 & = & 0 \\ & -x_1 & \leq & 0 \\ & x_2 - 6 & \leq & 0 \\ & -2x_2 + 3 & \leq & 0. \end{array}$$

  a) Solve the problem graphically. Draw a picture of the feasible region of the problem and draw also the contour lines of the objective function. (2p)
  b) Present the necessary Karush-Kuhn-Tucker (KKT) conditions and see if the solution satisfies these conditions. (4p)

The exam continues next page

- You are the production manager for car manufacturing at Teddy's Four-Wheel, Inc. Currently, the collection of sold cars consists only of one model, Winnie. The production of a Winnie requires 2 tonnes of steel and 100 hours of work. The company has also an option to extend its operations and to begin to manufacture new sport car model named Bear. A Bear requires 1.5 tonnes of steel and 150 hours of work to be manufactured. In addition, the production of Bers requires an investment to a new production line. The production line costs the same amount as the profit from selling 1000 Bear cars, and the life of the production line would be 10 years. Each week the company has 24 tonnes of steel and 1200 hours of work in use. The profit from a Bear model car is double compared to the profit from selling a Winnie model car.

  a) What kind of weekly production would maximize the profits of the company? Formulate the problem as *linear integer problem*. You do not need to solve the problem. (4p)
  b) You plan to solve the problem with Branch-and-Bound algorithm. Present the general description of the algorithm. (2p)

- Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

  a) For a linear programming (LP) problem, there is always either exactly one solution or no solution at all.
  b) It is possible to formulate such an optimization problem, which has exactly two solutions.
  c) Consider a linear programming (LP) problem. If the solution of the primal problem is unbounded, then the dual problem has no feasible solution.