AALTO UNIVERSITY SCHOOL OF SCIENCE
Department of Mathematics and Systems Analysis

MS-C2105 Introduction to optimization Harri Ehtamo/Ilmari Pärnänen

Exam, 8.4.2015

1. Briefly define the following terms:
   - binary choice
   - set-covering problem
   - subtour
   - Lagrangian function
   - non-feasible basic solution
   - simplex iteration

2. Use tabular Simplex algorithm in the following problem.

$$\begin{array}{rrcl} \min & x_1 - 2x_2 & & \\ \text{s.t.} & x_1 + x_2 & \geq & 2 \\ & -x_1 + x_2 & \geq & 1 \\ & x_2 & \leq & 3 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem with Simplex algorithm. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

3. Consider a problem

$$\begin{array}{lllll} \max & 9x_1 + 5x_2 & & \\ \text{s.t.} & 4x_1 + 9x_2 & \leq & 35 \\ & x_1 & \leq & 6 \\ & x_1 - 3x_2 & \geq & 1 \\ & 3x_1 + 2x_2 & \leq & 2 \\ & x_1, x_2 & \in & \mathrm{Z}_+ \cup \{0\} \end{array}$$

   a) Solve the LP relaxation of the problem graphically. (1p)
   b) Determine the solution to the problem with Branch & Bound algorithm. Solve subproblems graphically. (3p)
   c) Draw the course of your solution in tree form, and justify with it that the solution you got is the best possible integer solution to the original problem. (2p)

4. Mantel Ltd. produces a toy cars, whose final assembly must include four wheels and two seats. The factory producing the parts operates three shifts a day. The followind table provides the amounts produced of each part in the three shifts.

	Unites produced per run	
Shift	Wheels	Seats
1	500	300
2	600	280
3	640	360

Ideally, the number of wheels produced is exactly twice that of the number of seats. However, because production rates vary from shift to shift, exact balance in production may not be even possible. Thus Mantel is interested in determining the number of production runs in each shift that minimizes the imbalance in the production of the parts. The capacity limitations restrict the number of runs to between 4 and 5 for shift 1, 10 and 20 for shift 2, and 3 and 5 for shift 3.

The company has the necessary software to solve the problem, but writing the problem properly to computer causes problems. The manager asks you, a summer trainee who has studied the Introduction to optimization course, to help in the matter. Formulate the manufacturing of the parts as a goal programming model.

5. Consider a problem

$$
\begin{array}{lrcl}
\min & (x_1 - a)^2 + (x_2 - b)^2 & & \\
\text{s.t.} & -4x_1 + 4x_2 & \leq & 12 \\
 & 5x_1 + 2x_2 & \leq & 20 \\
 & x_1, x_2 & \geq & 0,
\end{array}
$$

where i) $a = 2, b = 2$ and ii) $a = 4, b = 6$.

a) Solve cases i) and ii) graphically. Draw a picture of the feasible region of the problem and draw also the contour lines of the objective function. (2p)
b) In both cases, present the necessary Karush-Kuhn-Tucker (KKT) conditions and see if the solution satisfies these conditions. (4p)

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MS-C2105 Introduction to optimization Harri Ehtamo/Markus Mattila

Exam, 8.4.2015

1. Briefly define the following terms:
   - slack variable
   - binary choice
   - subtour
   - function f is convex
   - portfolio optimization
   - shadow price

2. Use tabular Simplex algorithm in the following problem.

$$\begin{array}{lrcl} \max & 3x_1 - 2x_2 & & \\ \text{s.e.} & x_1 + x_2 & \geq & 2 \\ & 3x_1 - x_2 & \leq & 8 \\ & -x_1 + 4x_2 & \leq & 12 \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem using the M method. (3p)
   c) Draw a picture of the feasible region of the problem and show how Simplex algorithm progresses in the region. (2p)

3. Consider a problem

$$\begin{array}{lllc} \min & (x_1 - 5)^2 + (x_2 - 3)^2 & & \\ \text{s.e.} & x_2^2 - x_1 & \leq & 0 \\ & 2x_2 - x_1 & = & 0 \\ & x_1, x_2 & \geq & 0. \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and also draw the contour lines of the objective function. (2p)
   b) Present the necessary Karush-Kuhn-Tucker (KKT) conditions and see if the solution satisfies these conditions. (4p)

4. Mantel Ltd. produces a toy cars, whose final assembly must include four wheels and two seats. The factory producing the parts operates three shifts a day. The followind table provides the amounts produced of each part in the three shifts.

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The company has the necessary software to solve the problem, but writing the problem properly to computer causes problems. The manager asks you, a summer trainee who has studied the Introduction to optimization course, to help in the matter. Formulate the manufacturing of the parts as a goal programming model.

5. Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

a) For a linear programming (LP) problem, there is always either exactly one solution or no solution at all.
b) It is possible to formulate such an optimization problem, which has exactly two solutions.
c) Consider a linear programming (LP) problem. If the solution of the primal problem is unbounded, then the dual problem has no feasible solution.

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Exam, December 14th 2015

1. Use tabular Simplex algorithm in the following problem.

$$\begin{array}{rrcr} \max & 2x_1 + x_2 & & \\ \text{s.t.} & 2x_1 - x_2 & \leq & 6 \\ & x_1 + 2x_2 & \leq & 10 \\ & x_1 - x_2 & \geq & \text{-3} \\ & x_1, x_2 & \geq & 0 \end{array}$$

   a) Transform the linear problem into the standard form. (1p)
   b) Solve LP problem with Simplex algorithm. (3p)
   c) Draw a picture of the feasible region of the problem and how Simplex algorithm progresses in the region. (2p)

2. Consider a problem

$$\begin{array}{lllr} \min & (x_1 - 5)^2 + (x_2 - 4)^2 & & \\ \text{s.e.} & x_1^2 - 4x_1 - x_2 + 5 & \leq & 0 \\ & 2x_1 + 3x_2 - 12 & = & 0 \\ & -x_1 & \leq & 0 \\ & x_2 - 6 & \leq & 0 \\ & -2x_2 + 3 & \leq & 0. \end{array}$$

   a) Solve the problem graphically. Draw a picture of the feasible region of the problem and draw also the contour lines of the objective function. (2p)
   b) Present the necessary Karush-Kuhn-Tucker (KKT) conditions and see if the solution satisfies these conditions. (4p)

The exam continues next page

3. You are the production manager for car manufacturing at Teddy's Four-Wheel, Inc. Currently, the collection of sold cars consists only of one model, Winnie. The production of a Winnie requires 2 tonnes of steel and 100 hours of work. The company has also an option to extend its operations and to begin to manufacture new sport car model named Bear. A Bear requires 1.5 tonnes of steel and 150 hours of work to be manufactured. In addition, the production of Bers requires an investment to a new production line. The production line costs the same amount as the profit from selling 1000 Bear cars, and the life of the production line would be 10 years. Each week the company has 24 tonnes of steel and 1200 hours of work in use. The profit from a Bear model car is double compared to the profit from selling a Winnie model car.

   a) What kind of weekly production would maximize the profits of the company? Formulate the problem as *linear integer problem*. You do not need to solve the problem. (4p)
   b) You plan to solve the problem with Branch-and-Bound algorithm. Present the general description of the algorithm. (2p)

4. Briefly define the following terms.

   a) Corner point (1p)
   b) Shadow price (1p)
   c) Binary variable (1p)
   d) Lagrangian function (1p)
   e) Efficient, or Pareto optimal solution (1p)
   f) Subtour (1p)

5. Which of the following claims are true and which are false? Justify your choice. For a correct answer you gain +1 point and for right justification +1 point. For a false answer you get -2 points and an empty answer is worth of 0 points.

   a) For a linear programming (LP) problem, there is always either exactly one solution or no solution at all.
   b) It is possible to formulate such an optimization problem, which has exactly two solutions.
   c) Consider a linear programming (LP) problem. If the solution of the primal problem is unbounded, then the dual problem has no feasible solution.