

Q1

Short description (max 150 words): Explain the terms *manipulated variable*, *controlled variable* and *disturbance variable*.

Q2

Short description (max 75 words): In the context of controller design, what means the *derivative kick*? Which types of controllers it may affect?

Q3

Short description (max 75 words): What means *feedforward control*?

Q4

Essay (max 600 words): What are the working principles of analog and digital signals? How the use of digital signals has transformed the field of process control?

Q5

Short description (max 150 words): What is the difference between the *process gain*  $K_p$ , *controller gain*  $K_c$ , and *sensor gain*  $K_m$ ?

# Process control and automation (CHEM-C2140)

## Exam, part 2: mathematical problems

April 6, 2020

Q6) The temperature inside the jacketed continuous process reactor (see Fig 1) is controlled by manipulating the volumetric flow rate of the cooling medium  $q_c$ . The cooling medium is assumed to remove heat from the tank at the rate  $Q$  that is linearly proportional to its volumetric flow rate  $q_c$ :

$$Q = kq_c$$

where  $k$  is a constant representing the effectiveness of the heat transfer (unit:  $[\text{J}/\text{m}^3]$ ). The chemical process inside the tank generates heat at the rate of  $E_r$  (unit:  $[\text{J}/\text{s}]$ ). However, this rate is not constant but may vary over time. The inlet flow temperature  $T_{\text{in}}$  is constant.

Derive a physics-based model for the temperature  $T$  in the tank as a function of

- volumetric inlet flow rate  $q_{\text{in}}$ ,
- volumetric flow rate of the cooling medium  $q_c$ ,
- process fluid temperature at the inlet  $T_{\text{in}}$ ,
- mass of process fluid in the tank  $m$ ,
- density of the process fluid  $\rho$ ,
- heat generation rate of the reaction  $E_r$ ,
- heat transfer constant  $k$ , and
- specific heat capacity of the process fluid  $c$ .

The process liquid mass  $m$  inside the reactor is held constant. The reactor is also well-mixed. State also other assumptions that are needed. Follow the steps of the systematic model development (discussed on Lectures 1 & 3).

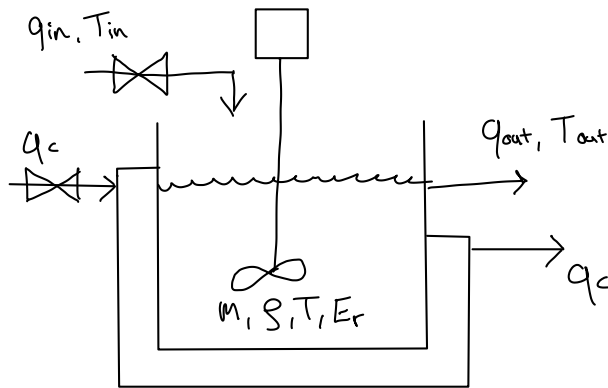


Figure 1: Jacketed continuous process reactor.

(8p)

Q7) Let us consider the differential equation

$$-4 \frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} - y(t) = 3$$

with initial conditions  $y(0) = 0$ ,  $y'(0) = \frac{3}{2}$ . Solve the differential equation using the Laplace transform. In other words, express  $y(t)$  as an algebraic equation that does not involve any derivatives of  $y(t)$ .

(6p)

Q8) Sketch by hand the output response  $y(t)$  of the FOPDT model when the input variable  $u(t)$  is that shown in Fig. 2. The FOPDT model has the process gain of  $K_p = -5$ , time coefficient of  $\tau_p = 1$  s, and dead time of  $\theta_p = 2.5$  s. At  $t = 0$  s, the output response is at a steady-state value of  $y(0) = 50$ . Make the sketch for the range of  $[0, 20]$  s.

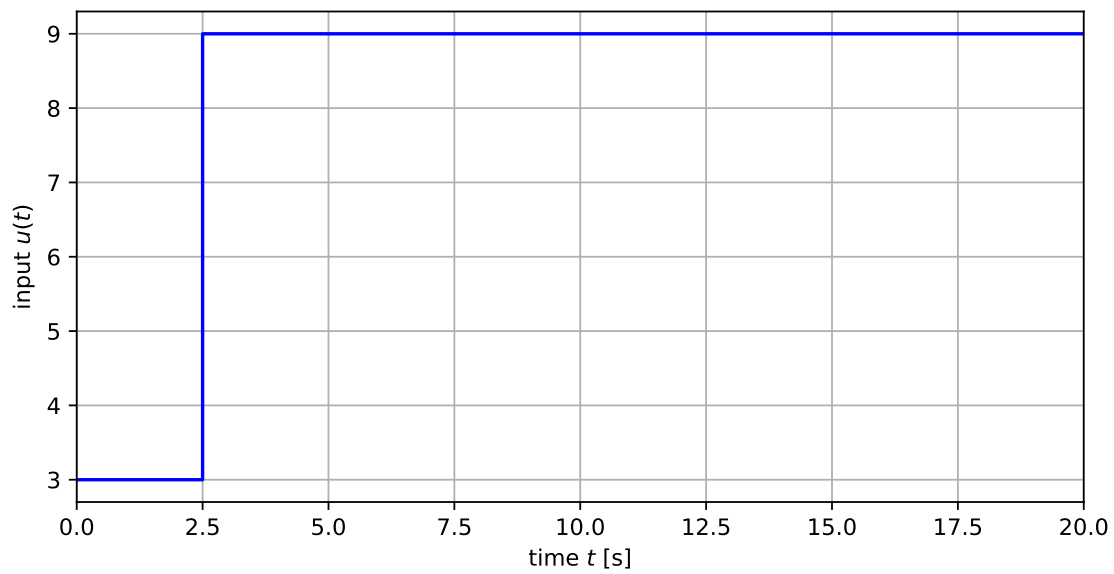


Figure 2: The input variable  $u(t)$  during the time window  $t \in [0, 20]$  s.

(3p)

Q9) Let us consider the liquid tank shown in Fig. 3. The volumetric outlet flow rate  $q_{out}$  from the tank is linearly proportional to the liquid level  $h$  in the tank

$$q_{out} = rh,$$

where  $r$  is a constant. The physics-based model for the tank level  $h$  is

$$\frac{dh}{dt} = \frac{1}{A} (q_{in} - rh),$$

where  $q_{in}$  is the volumetric inlet flow rate,  $\rho$  is the density of the liquid, and  $A$  is the cross-sectional area of the tank. At  $t = 0$ , the tank is empty. Therefore, the initial condition is:  $h(0) = 0$ .

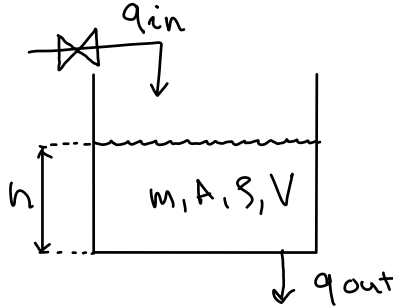


Figure 3: A liquid tank with a hole.

- a) Derive the transfer function  $G(s) = H(s)/Q_{in}(s)$  (in which  $H(s)$  and  $Q_{in}(s)$  are the liquid level and the volumetric inlet flow rate, respectively, in the Laplace domain) based on the physics-based model.
- b) Determine the process gain  $K_c$  based on the transfer function.

(4p)