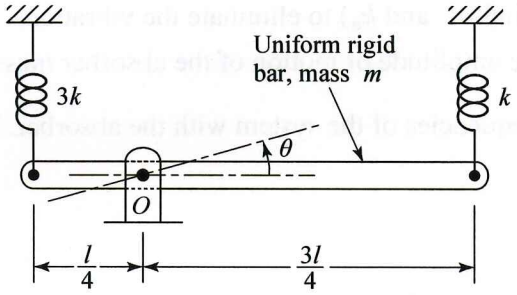
A? Problem 1 (10 pts)

Derive the equation of motion and find the natural angular frequency ω_n for the system shown below. The rotational inertia of the bar around point O is $J = \frac{7}{48}ml^2$. Assume small rotations.



A? Problem 2 (10 pts)

A spring-mass-damper system is subjected to a harmonic force. The amplitude is found to be 20 mm at resonance and 10 mm at a frequency 0.75 times the resonant frequency. Find the damping ratio ζ of the system.

A? Problem 3 (10 pts)

The base of a damped spring-mass system, with $m = 25\,\text{kg}$ and $k = 2500\,\text{N/m}$ is subjected to a harmonic excitation $y(t) = Y_0 \cos \omega t$. The amplitude of the mass is 0.05 m when the base is excited at the natural frequency of the system with $Y_0 = 0.01\,\text{m}$. Determine the damping constant c of the system.

A? Problem 4 (10 pts)

A variable-speed electric motor, having an unbalance, is mounted on an isolator. As the speed of the motor is increased from zero, the amplitudes of vibration of the motor are observed to be 1.4 mm at resonance ($r = 1$) and 0.4 mm beyond resonance ($r \to \infty$). Find the damping ratio ζ of the isolator.

A? Problem 5 (15 pts)

A diesel engine, weighing 3000 N, is supported on a pedestal mount. It has been observed that the engine is vibrating heavily (resonance) when operating at 6000 rpm.

(a) Design an absorber (find m_a and k_a) to eliminate the vibrations. The magnitude of the exciting force is 250 N, and the amplitude of motion of the absorber mass is limited to 2 mm.

(b) Find the resonance frequencies of the system with the absorber. Express your result in rpm.

A? Problem 6 (15 pts)

Two identical pendulums, each with mass m and length l, are connected by a spring of stiffness k at a distance d from the fixed end, as shown below. Recall that the rotational inertia of a single pendulum is $J = ml^2$.

(a) Derive the equations of motion using Lagrange's equation. Assume that rotations are small.

(b) Find the eigenvalues and eigenvectors of the system given that the mass and stiffness matrices are:

$$\boldsymbol{M} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \quad \text{and} \quad \boldsymbol{K} = \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}$$