Aalto University
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Exam, Monday December 7th 2020, 09:00 - 13:00

Complex Analysis, MS-C1300

Motivate your answers. Only giving answers gives no points.

See exam instructions here:
`https://mycourses.aalto.fi/mod/page/view.php?id=665605`

(1) (a) Give an example of a function $f\colon \mathbb{C} \to \mathbb{C}$ that is *not* analytic. (1p)

(b) Let $z = x + iy \in \mathbb{C}$. Is

$$f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$$

analytic? Justify your answer. (2p)

(c) Let $z = x + iy \in \mathbb{C}$. Is

$$f(z) = \frac{x}{x^2 + y^2} - i\frac{y}{x^2 + y^2}$$

analytic in $\mathbb{C} \setminus \{0\}$? Justify your answer. (3p)

(2) (a) Let $a \neq 0$ and $b \neq 0$ be complex numbers. Find the Taylor series of

$$f(z) = \frac{1}{az + b}$$

around $z_0 = 0$. Determine the radius of convergence ρ for the series. (2p)

(b) Find the Taylor series of

$$f(z) = \begin{cases} \frac{\sin z}{z}, \text{ when } z \neq 0 \\ 1, \text{ when } z = 0 \end{cases}$$

around $z_0 = 0$. Determine the radius of convergence ρ for the series. (2p)

(c) Find the Laurent series of

$$f(z) = (z - 1)\sin(z^{-1})$$

in $\{z \in \mathbb{C}; 0 < |z| < \infty\}$. (2$p$)

(3) Let $a > 0$. Calculate

$$\int_0^\infty \frac{x^2}{(x^2 + a^2)^2}\, dx.$$

(6p)

(4) Let $D \subseteq \mathbb{C}$ be a domain and $f\colon D \to \mathbb{C}$ be an analytic function. Assume that there is a point $z_0 \in D$ such that

$$0 < |f(z_0)| \leq |f(z)|$$

for all $z \in D$. Prove that f is constant in D. (6p)

Good luck!