Matrix Algebra
MS-A0001
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Please follow the instruction given on the exam page. Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified.

PROBLEM 1 Let

$$A = \begin{pmatrix} 6 & 16 & 21 \\ 10 & 0 & -15 \\ 4 & 4 & 14 \end{pmatrix}, \quad b = \begin{pmatrix} 10 \\ -50 \\ 40 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ -5 \\ 4 \end{pmatrix}.$$

a) Is x the solution of the equation $Ax = b$?

b) Is x an eigenvector of the matrix A?

PROBLEM 2 Solve the linear system $Ax = b$ for all admissible values of $\beta \in \mathbb{R}$, where

$$A = \begin{pmatrix} 2 & 3 & -1 & 4 \\ 3 & -1 & 0 & 1 \\ 1 & -4 & 1 & -2 \\ 2 & -2 & -2 & 5 \end{pmatrix}, \quad b = \begin{pmatrix} 16 \\ -5 \\ -22 \\ \beta \end{pmatrix}.$$

PROBLEM 3 (a) Find the LU-decomposition of

$$A = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

(b) Compute $\det(A)$.

PROBLEM 4 (a) Show that the vectors $a_1 = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$, $a_2 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}^{\mathrm{T}}$, $a_3 = \begin{pmatrix} 1 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$ form a basis of $\mathbb{R}^3$. (b) Find the coordinates of the vector $x = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^{\mathrm{T}}$ in this basis.

PROBLEM 5 (a) Find the angle between the eigenvectors of the matrix

$$\begin{pmatrix} 1 & 1 \\ 0 & 1+t \end{pmatrix}$$

as a function of the parameter t. (b) What is the relation between the angle and the linear independence of the eigenvectors?