## A. Expected utility theory ( 6 p )

The below table shows four decision alternatives some of whose outcomes are uncertain. Mr. Rational prefers more to less and his preferences follow the axioms of utility theory. He prefers A to B.

| A |  | B |  | C |  | D |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| outcome <br> (€) | probability | outcome <br> (€) | probability | outcome <br> (€) | probability | outcome <br> (€) | probability |
|  |  | 50 | 0.4 | 50 | $p_{50}^{C}$ | 50 | $p_{50}^{D}$ |
| 70 | 1 |  |  | 70 | $p_{70}^{C}$ | 70 | $p_{70}^{D}$ |
|  |  | 100 | 0.6 | 100 | $p_{100}^{C}$ | 100 | $p_{100}^{D}$ |

1. Which one of alternatives $C$ and $D$ does Mr. Rational prefer? (1p)
2. C
3. D
4. cannot be determined with the information available
5. Which risk attitude is possible for Mr. Rational? (1p)
6. risk neutral
7. risk averse
8. risk seeking
9. cannot be determined with the information available
10. What is the certainty equivalent of alternative $A$ ? (1p)

Round your answer to full euros.
4. The certainty equivalent of alternative $B$ is ... (1p)

1. less than 50 euros
2. in $[50,70)$
3. in $[70,100)$
4. at least 100 euros
5. cannot be determined with the information available
6. Does alternative $D$ dominate alternative $C$ in the sense of first-degree stochastic dominance? ( $\mathbf{1 / 2} \mathbf{p}$ )
7. yes
8. no
9. cannot be determined with the information available
10. Does alternative $\mathbf{C}$ dominate alternative $\mathbf{D}$ in the sense of first-degree stochastic dominance? ( $\mathbf{1 / 2} \mathbf{p}$ )
11. yes
12. no
13. cannot be determined with the information available
14. Does alternative $\mathbf{C}$ dominate alternative $\mathbf{D}$ in the sense of second-degree stochastic dominance? ( $\mathbf{1 / 2} \mathbf{p}$ )
15. yes
16. no
17. cannot be determined with the information available
18. Does alternative $\mathbf{D}$ dominate alternative $\mathbf{C}$ in the sense of second-degree stochastic dominance? ( $\mathbf{1 / 2} \mathbf{p}$ )
19. yes
20. no
21. cannot be determined with the information available

## B. Decision trees (6p)

A risk neutral manufacturing company must decide whether to manufacture a component part for a product or purchase this part from a supplier. The revenues resulting from the two decision alternatives depend on whether the demand for the product is high, medium or low as follows:

|  | High | Medium | Low |
| :--- | :---: | :---: | :---: |
| manufacture | $r_{H}$ | $r_{M}$ | $r_{L}$ |
| purchase | $70000 €$ | $45000 €$ | $10000 €$ |

The company estimates the probabilities of these demands to be $p(H), p(M)$ and $p(L)$, respectively.

To obtain more information about the future demand, the company can buy a market study. This study costs $C$ euros. The result of the market study can be favorable or unfavorable, and the probabilities of these results depend on the demand as follows:

|  | High | Medium | Low |
| :--- | :---: | :---: | :---: |
| favorable | 0.8 | 0.4 | 0.1 |
| unfavorable | 0.2 | 0.6 | 0.9 |

9. What is the probability that the result of the market study is favorable? (1p)

Give your answer with the precision of two decimals.
10. The optimal strategy is to (1p)

1. Manufacture the component without buying the market study
2. Purchase the component without buying the market study
3. Buy the market study and purchase the component if the market study says "Favorable"; otherwise not.
4. Buy the market study and manufacture the component if the market study says "Favorable"; otherwise not.
5. What is the probability of High demand, if the result of the market study is 'unfavorable'? (1p)

Give your answer with the precision of two decimals.
12. What is the expected revenue of optimal decisions? (2p)

Round your answer to full euros.
13. How much should the cost of the study $(C)$ be so that the company would be indifferent between buying the market study and not buying it? (1p)

Round your answer to full euros.

## C. Multi-objective optimization and multi-attribute value theory (4p)

Riikka is selecting a road pavement project. Renewing the pavement improves the quality of the road. The decision alternatives are described with three technical measures (attributes) which measure the current condition of the roads (alternatives): the worse the condition, the higher the technical measure describing the attribute and consequently the higher the value for renewing the pavement. All alternatives are of same length and they do not differ in average daily traffic or cost. In questions 16-17, an additive value model describes Riikka's preferences. Attribute-specific value functions are linear. The 8 alternatives are described in the below table.

| alternative | grooves | holes | other road safety issues |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 4 | 1 |
| 2 | 2 | 3 | 1 |
| 3 | 4 | 4 | 1 |
| 4 | 3 | 2 | 1 |
| 5 | 2 | 7 | 1 |
| 6 | 9 | 0 | 1 |
| 7 | 4 | 3 | 1 |
| 8 | 1 | 7 | 1 |

14. Which alternatives could be found with the weighted max-norm approach? (1p) Answer with the alternatives' numbers in a increasing order, do not use space; e.g. answer 123 corresponds to alternatives 1, 2 and 3)
15. Which alternatives are pareto optimal? (1p)

Answer with the alternatives' numbers in a increasing order, do not use space; e.g. answer 123 corresponds to alternatives 1, 2 and 3)
16. Riikka now thinks that in modeling pavement need, a unit increase in attribute holes is at least as valuable as a unit increase in attribute grooves. Which ones are nondominated? (1p)
Answer with the alternatives' numbers in a increasing order, do not use space; e.g. answer 123 corresponds to alternatives 1, 2 and 3)
17. Riikka specifies her preferences: an increase of 2 units in holes is as valuable as an increase of 3 units in grooves. Which alternative would Riikka choose? (1p)

1. 1
2. 2
3. 3
4. 4
5. 5
6. 6
7. 7
8. 8
9. cannot be determined with the available information
