Course Name: Production Planning and Optimization
Course Code: CHEM-E7151
Final Exam, 19.10.2020, 09.00-13.00
Exam Method: MyCourses Assignment activity with a time restriction.
Write your full name and student number on the top of your answer sheets.
Submit your answers to the Submission folder named "Final exam".
The exam has a total of $\mathbf{2 0}$ points.

## Question 1. (2 points)

Metallica Manufacturing Company produces three grades (low, medium, and high) of aluminum at three mills. It costs the company 900 euros per day to operate mill 1,1000 euros per day to operate mill 2, and 1250 euros per day to operate mill 3 . Each mill has a different production capacity in tons per day for each grade, as shown in Table 1. The company has contracted with a manufacturing firm to supply at least 35 tons of low-grade aluminum, 20 tons of medium-grade aluminum, and 22 tons of high-grade aluminum. The company wants to determine the number of days of operating each mill to meet the contract demand requirements at the minimum cost.
Table 1. Production capacity of each mill in tons per day for producing each aluminum grade

| Aluminum Grade | Mill |  |  |
| :--- | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| Low | 5 | 11 | 9 |
| Medium | 3 | 3 | 7 |
| High | 7 | 4 | 6 |

## a) Briefly answer:

- Using what type of model can this problem be formulated? ( 0.25 points)
- Which approach can be used to find a solution for this problem? ( 0.25 points)
- In what way does the selected method find the optimal solution? ( 0.25 points)
b) Formulate the given problem. Do not solve it! (1.25 points)


## Question 2. (3 points)

Chemco Corporation produces two chemicals namely Ethylene and Butadiene. Ethylene sells for 9 euros per kilograms (kg), and Butadiene sells for 7 euros per kg. Production of Ethylene requires 12 kg of raw material and 4 hours of skilled labor. Production of Butadiene requires 4 kg of raw material and 8 hours of skilled labor. A total of 40 hours of skilled labor and 60 kg of raw material are available.

Use the Simplex method to maximize the Chemco's sales revenues and find the optimal amount of chemicals to be produced.

## Question 3. (4 points)

A company is going to introduce a new product into a competitive market and is currently planning its marketing strategy. The decision has been made to introduce the product in three stores. To test its marketing strategy, initially, only five batches of the respective product are going to be manufactured. The estimated probability distribution of potential sales of these batches differs among the three stores. Therefore, the owner wants to know how to allocate five batches to the three stores to maximize the profit. The owner does not wish to split batches between stores, and also willing to distribute no batch to stores if it does not contribute to profit. Table 2 gives the expected profit at each store when it is allocated various numbers of batches. Use Dynamic Programming to determine how many of the five batches should be assigned to each of the three stores to maximize the total expected profit.
Table 2. Estimated profit at each store from selling the batches

| Batch of Product | Profit (in 1000 euros) |  |  |
| :---: | :---: | :---: | :---: |
|  | Store 1 | Store 2 | Store 3 |
| $\mathbf{0}$ | 0 | 0 | 0 |
| $\mathbf{1}$ | 4 | 6 | 5 |
| $\mathbf{2}$ | 9 | 11 | 9 |
| $\mathbf{3}$ | 13 | 15 | 14 |
| $\mathbf{4}$ | 18 | 19 | 17 |
| $\mathbf{5}$ | 20 | 22 | 21 |

## Question 4. (5 points)

A specialty chemical plant produces two products X and Y . Producing a unit of product X requires 3 hours of labor, 35 kg of raw material A and 10 kg of raw material B. Producing a unit of product Y requires 2 hours of labor, 15 kg of raw material A and 15 kg of raw material B . The total amount of available labor is 100 hours and there is 400 kg of raw material A and 500 of raw material B . The profit is $450 \mathrm{EUR} /$ unit for product X and $200 \mathrm{EUR} /$ unit for product Y . The company wants to know the optimal production strategy that maximizes the total profit.
a) Formulate the problem as an integer programming problem. (2 points)
b) Solve this model by using the graphical method without integer restrictions (relaxed LP problem). What special can you observe about this particular problem? (1 point)
c) Perform the Branch and Bound algorithm and compare the obtained optimum solution with the relaxed linear programming model solved in part b). You can do it either manually or using e.g. GAMS (in which case submit the code). (2 points)

Question 5. (4 points)
A beverage plant produces different juices, in this case they are A: cranberry, B: blueberry, C: a mix of cranberry \& blueberry, and D: orange juice. The process is done on one mixer and in order to avoid contamination and bacterial growth the mixer must be sterilized after each batch (1 hour). In addition to this, there are some setup activities needed that depend on the sequence and are shown in Table 3 below. The processing times of a batch are shown in Table 4. One batch of each type of juice should be produced.

Table 3. Changeover time between the products (given in minutes)

| Juice | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| A | 0 | 30 | 15 | 60 |
| B | 25 | 0 | 20 | 60 |
| C | 30 | 30 | 0 | 60 |
| D | 90 | 90 | 70 | 0 |

Table 4. Batch processing times of juice products (given in minutes)

| Juice | Time |
| :---: | :---: |
| A | 120 |
| B | 120 |
| C | 140 |
| D | 100 |

a) Formulate the scheduling problem minimizing the makespan. (1 point)
b) Define the sets and data needed. (1 point)
c) Solve it with GAMS. (2 points)

## Question 6. (2 points)

Let us consider a three-stage batch process with a demand to produce five tasks. Stage-specific durations of the tasks are listed in Table 5. Each stage has only one processing unit, which can process only one task at the time. No changeover times between tasks are considered. The processing of a task at Stages 2 and 3 can start only after its processing has finished at the earlier stage.
a) Use the 'shortest processing time first' heuristic to define the dispatching sequence for the tasks. The name of the algorithm refers to the processing time at the first stage. ( 0.5 points)
b) The dispatching is started at $t=0 \mathrm{~h}$. The processing of tasks at Stages 1 to 3 are started as early as possible. Draw the Gantt chart of the schedule. (1 point)
c) Determine the makespan of the tasks using the Gantt chart. ( 0.5 points)

Table 5.

| Task | Processing duration at <br> Stage 1 $[\mathbf{h}]$ | Processing duration at <br> Stage 2 $\mathbf{[ h ]}$ | Processing duration at <br> Stage 3 $\mathbf{[ h} \mathbf{]}$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 4 | 3 | 7 |
| $\mathbf{2}$ | 7 | 3 | 4 |
| $\mathbf{3}$ | 2 | 3 | 4 |
| $\mathbf{4}$ | 5 | 3 | 4 |

