CS-E3220 Declarative Programming

Examination, December 14, 2020

As in all examinations, you must answer the questions by yourself, without any outside help.

Using the material and software tools of the course is allowed.

Assignment 1 Formulas and normal forms. (Max. 5p)

Consider the propositional formula $a \vee (b \oplus c)$.

1. Give the truth table for the formula and all its subformulas.
2. Derive a logically equivalent formula that is in the conjunctive normal form.

Assignment 2 Conflict-Driven Clause Learning (Max. 5p)

Consider the CNF formula $(a \vee \neg f \vee \neg h) \wedge (b \vee c \vee \neg d) \wedge (c \vee \neg e) \wedge (d \vee e \vee f) \wedge (\neg f \vee \neg g) \wedge (g \vee h)$.

(a) Simulate the conflict-driven clause learning (CDCL) satisfiability algorithm by choosing decision variables in the alphabetic order $a, b, c, \ldots$, always setting the chosen variable to false, *until a conflict is reached*. Illustrate the resulting implication graph.

(b) Using the constructed implication graph, illustrate the "first UIP cut" and give the corresponding learned clause.

Assignment 3 CSPs and Arc Consistency (Max. 5p) Consider the sum constraint $2x_1 + 2x_2 + x_3 + x_4 + 2x_5 = z$ when the domains are $D(x_1) = \{1, 2\}$, $D(x_2) = \{1, 3\}$, $D(x_3) = \{2, 3\}$, $D(x_4) = \{1, 3\}$, $D(x_5) = \{0, 1, 2\}$, and $D(z) = \{1, 3, 5, 9\}$.

Answer the following questions by using the dynamic programming approach described in the lecture slides. Illustrate the relevant parts of the graph presentation in your answer.

- Does the constraint have solutions? If it has, give one. If it does not, argue why this is the case.
- Is the constraint arc-consistent? If it is, explain why this is the case. If it is not, filter the constraint to be arc-consistent, and give the resulting new domains.

Assignment 4 Binary Decision Diagrams (Max. 5p)

1. Construct an OBDD for the Boolean function that yields *true* if and only if A and D have the same value and B and C have the same value. The variable ordering is A, B, C, D.
2. Annotate each node of your OBDD with the corresponding model-count.

Assignment 5 Modal logics (Max. 5p)

For each of the following formulas, give a model in which the formula is true in w_0, or argue why the formula cannot be true in any state/world.

1. $a \wedge (\Box \neg a) \wedge (\Box \Box a)$
2. $(A\mathcal{G}E\mathcal{F}\neg a) \wedge E\mathcal{G}a$

Assignment 6 (Max. 5p)

Explain why every possible directed graph, consisting of a set of $\leq 2^n$ states and a set of directed edges between states, can be expressed in terms of a set X of n Boolean state variables and a transition formula Φ over X and X' (where the latter has the "next state" variants x' of the state variables $x \in X$). How do you systematically construct Φ for any such given graph?

NB: Assignments continue on the other side of the question sheet!

Assignment 7 State-space search with propositional logic (Max. 5p)

Consider the following reachability problem in the propositional logic.

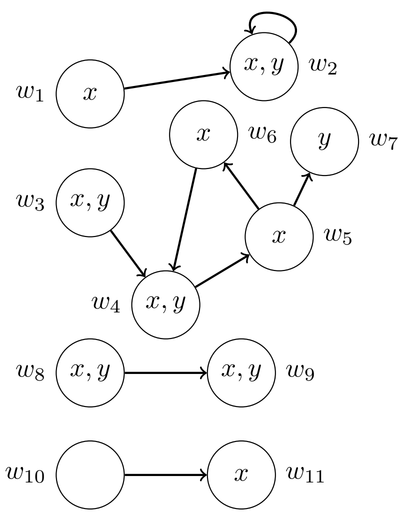
The states are encoded by the state variables a, b and c. The transitions are

- inverting the values of a and b and c (e.g. turning 101 to 010),
- inverting the values of a and c, and
- inverting the value of a.

Questions:

1. Give the formula for the transition relation that includes all 3 transitions.
2. Show how the image of the formula a w.r.t. the transition relation is computed.
3. How many steps does the symbolic breadth-first search algorithm run starting from $a \wedge \neg b \wedge c$? How many different states are reachable?

Assignment 8 (Max. 5p)



Run the CTL model-checking algorithm for the formula $\phi = EGEFy$ and the model on the left. For all of the relevant subformulas of ϕ, list which worlds will be labelled with that subformula.

Assignment 9 Satisfiability Modulo Theories (Max. 10p)

(a) Recall that T_{EUF} is the theory of uninterpreted functions and predicates (with built-in equality, as usual) and T_{LIA} is the theory of linear arithmetic over integers. Which of the following are true? Justify your answer by giving a model or a proof.

1. $(f(x) \not\approx x) \rightarrow (f(f(x)) \not\approx x)$, where f is a unary function, is T_{EUF}-valid.
2. $(y \geq x) \wedge (y < x + 3) \wedge (y \geq -2x) \rightarrow (x \geq -1)$ is T_{LIA}-valid. (5p)

(b) Show how the method presented in the material (constraint graphs and the Bellman–Ford algorithm) determines whether the following conjunction

$$(x-y \leq 3) \wedge (y-z \leq -1) \wedge (y-u \leq -2) \wedge (w-u \leq -3) \wedge (u-x \leq -1) \wedge (u-z \leq 2) \wedge (z-x \leq -2)$$

is IDL-satisfiable, where IDL is the integer difference logic. If the conjunction is IDL-unsatisfiable, find a negative cycle and give the corresponding IDL-unsatisfiable sub-conjunction. If it is IDL-satisfiable, give a model for the conjunction. (5p)

Assignment 10 At what time did you complete answering the questions?