Prediction and Time Series Analysis — 2020
Online exam
Answer to all the questions.

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1. True or False (4 p.)

   Determine whether the statement is true or false. You do not have to justify your answers. Simply state wether the statement is true or false. (Every correct answer +1 p., every wrong answer -1 p., no answer 0 p.)

   (a) In the context of linear regression, the traditional least-squares estimators are sensitive to outlying observations.
   (b) In the context of linear regression, conducting homoscedasticity testing is possible only if the residuals are normally distributed.
   (c) In exponential smoothing, the value of $x_{t+1}$ is predicted using a weighted sum of the previous observation $x_t, x_{t-1}, x_{t-2}, ...$
   (d) ARIMAX modeling is an excellent choice for long term forecasting.

2. Linear regression (3 p.)

   Consider the linear regression model

   $$y_i = \beta_0 + \beta_1 x_i + \epsilon_i.$$

   You have a sample and you estimate the parameters $\beta_0$ and $\beta_1$ using traditional least squares estimators. In your sample, you have two separate subgroups that have sample sizes $h$ and $n - h$, and you are worried about possible parameter instability.

   (a) Explain, step by step, how to apply a permutation test in testing parameter instability. (2 p.)
   (b) Give the null hypothesis of the test. (1/2 p.)
   (c) How would you proceed your modeling (what would you do), if the estimated $p$-value of the test statistic was very small? (1/2 p.)

3. Interval bootstrapping/block bootstrapping (4 p.)

   Assume that you have observed a time series $x_1, x_2, x_3, ..., x_{10263}$. Based on plotting the series and its estimated autocorrelation and partial autocorrelation -functions, you think that the observed series is an MA(1) process

$$x_t = \theta_1 \epsilon_{t-1} + \epsilon_t, \quad (\epsilon_t)_{t \in T} \sim WN(0, \sigma^2)$$

and you have estimated the parameter $\theta_1$. Your estimate is 0.56.

(a) Explain, step by step, how to construct a 95% bootstrap confidence interval for the parameter $\theta_1$. (3 p.)

(b) Explain how to use the confidence interval in testing (approximately) the significance of the parameter $\theta_1$? (1 p.)

4. Stationarity (3 p.)

   Let $x_t$ be a stochastic process such that, for all $t, \tau \in \mathbb{Z}$, we have that $E[x_t] = 0$, $E[x_t^2] = \sigma^2 < \infty$, $E[x_t x_{t-\tau}] = \gamma_\tau < \infty$, $E[x_t^4] = \kappa < \infty$, and $E[x_t^2 x_{t-\tau}^2] = \lambda_\tau < \infty$. Show that the process $y_t = x_t^2$ is weakly stationary.

5. ARMA modeling (4 p.)

   Assume that you observe a series $x_0, x_1, x_2, ..., x_{7305}$.

   (a) Based on plotting the series, you observe a linear trend. You manage to stationarize the process by taking a difference. The obtained differenced series is $z_1, z_2, ..., z_{7305}$. Give the elements $(z_t)$ of the stationarized process in terms of the elements of the original observed series. (1 p.)

   (b) Based on plotting the stationarized series and its estimated autocorrelation and partial autocorrelation -functions, you think that the stationarized series $z_1, z_2, ..., z_{7305}$ is an autoregressive process of order 2. You estimate the parameters of the process and the estimated values are $\phi_1$ and $\phi_2$. Give predictions for $z_{7306}, z_{7307}$ and $z_{7308}$. (2 p.)

   (c) Using the predictions for $z_{7306}, z_{7307}$ and $z_{7308}$, give predicted values for $x_{7306}, x_{7307}$ and $x_{7308}$. (1 p.)

6. Autocorrelations (6 p.)

   Figures 1 and 2 display the theoretical autocorrelation and partial autocorrelation -functions of six different processes. Answer to the following questions. You do not have to justify your answers. (Every correct answer +1 p., every wrong answer 0 p., no answer 0 p.)

(a) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an MA(2)-process?

(b) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an AR(3)-process?

(c) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is an ARMA(2,2)-process?

(d) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a MA(1)-process?

(e) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SAR(2)_3$-process?

(f) Which one of the processes (Series 1, 2, 3, 4, 5 or 6) is a $SMA(3)_3$-process?

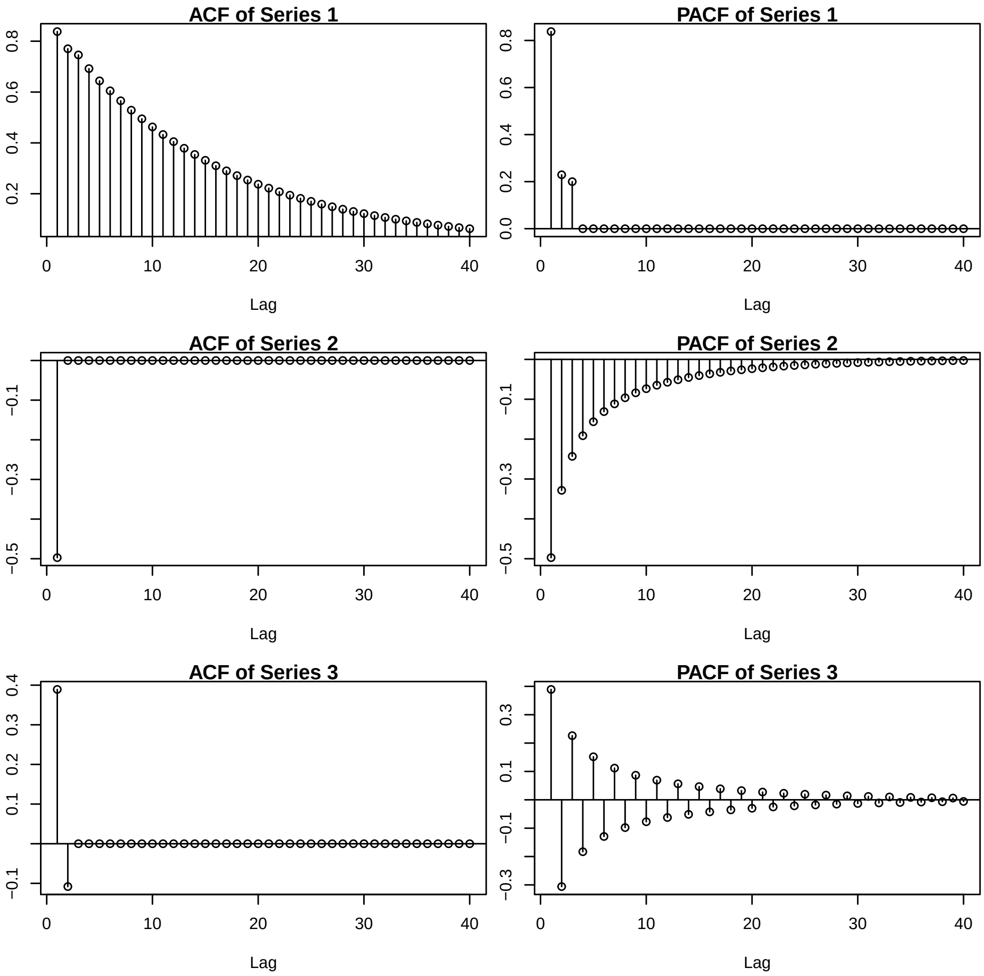
ACF of Series 1
PACF of Series 1
ACF of Series 2
PACF of Series 2
ACF of Series 3
PACF of Series 3
Lag
0
10
20
30
40
0.2
0.4
0.6
0.8
0.0
−0.1
−0.3
−0.5
0.1
0.3


Figure 1

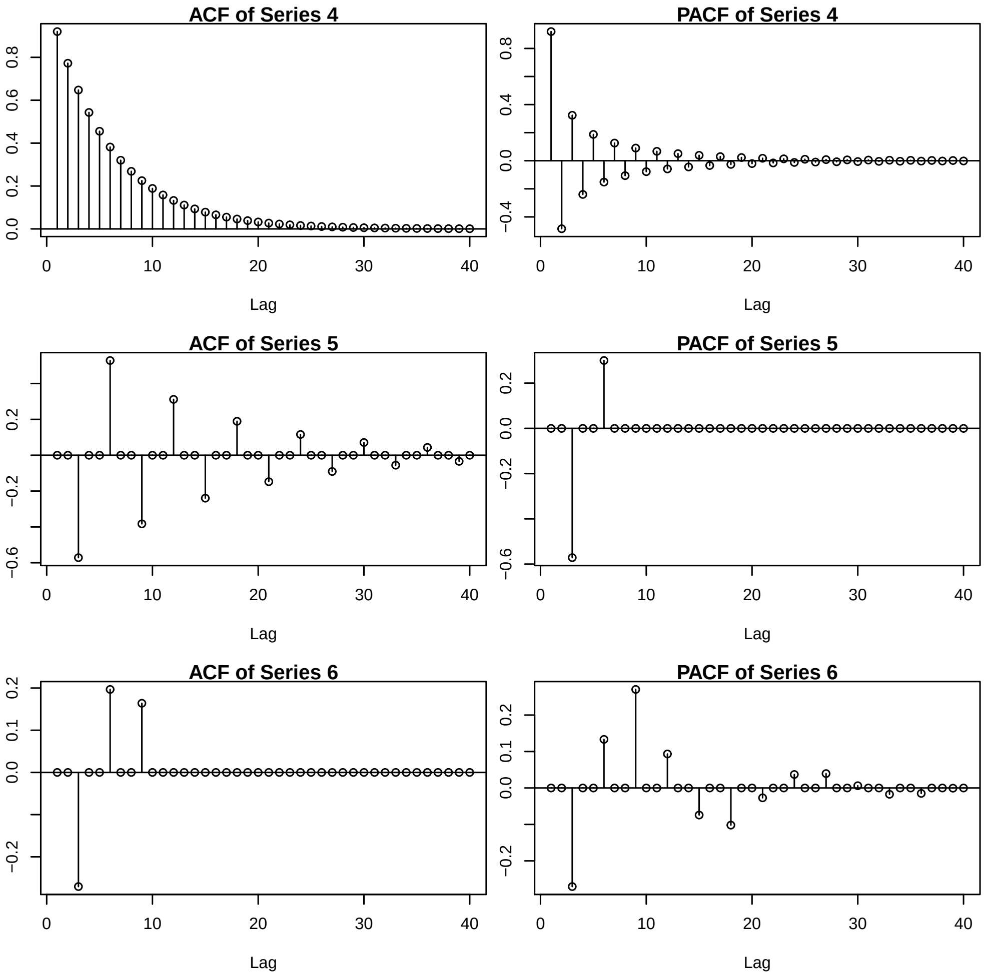
ACF of Series 4
PACF of Series 4
ACF of Series 5
PACF of Series 5
ACF of Series 6
PACF of Series 6
Lag
0.0
0.2
0.4
0.6
0.8
−0.4
−0.2
−0.6
0.1
0
10
20
30
40

Figure 2