CIV-E1020 - Mechanics of Beam and Frame Structures - duration: 3h

It is compulsory to solve only THREE (3) EXERCISES that you choose freely: only three best exercises (answers) will be graded even if the student solves four.

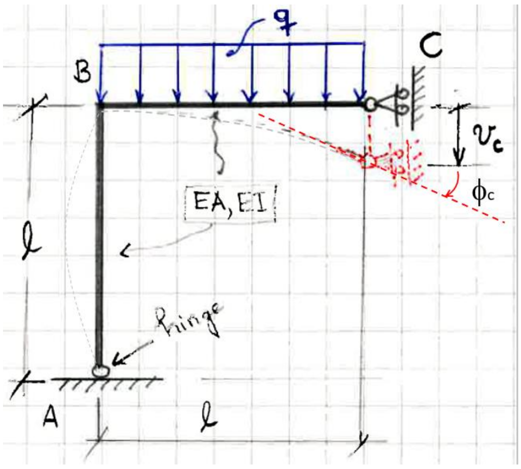
1) Results given without shown the logical steps needed to achieve them will be ignored even if correct.
2) Sequentially number (numeroi juoksevasti) your answer papers 1(*n*) … *n*(*n*), where *n* is the number of separate papers. On each of the papers, write readably your name, family name and student number.

Formulary enclosed with the questions

Examination 10.12.2020

The material is linear elastic in all the structures below

1. Use the dummy unit-load theorem (or method) and determine the rotation ϕ_c at the roller *C*. *Account for both the effects of bending and axial forces*. [5p]

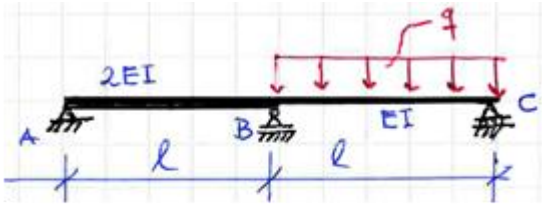


Grading 3 obligatory exercises		GRADE
Points in this exam		
14.25	15	5
12.75	13.5	4
9.75	12	3
8.25	9	2
6	7.5	1
< 6	fail	0

2. Use the general force method and

a) determine the bending moment at the mid-support *B* [3p] and draw accurately the bending moment diagram [1p]. *Account only for effects of bending*

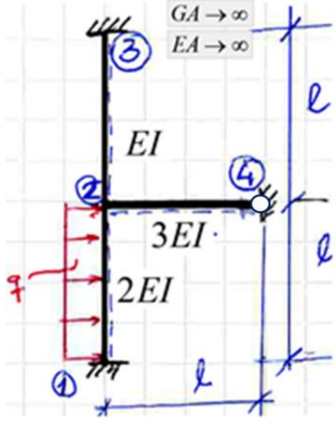
b) Determine the support reaction at *A* (value and direction). [1p].



3. Use Slope-Deflection Method and

a) **determine the bending moment** at the clamping support *1* and [4 points]

b) determine and draw accurately the bending moment diagram [1 point]



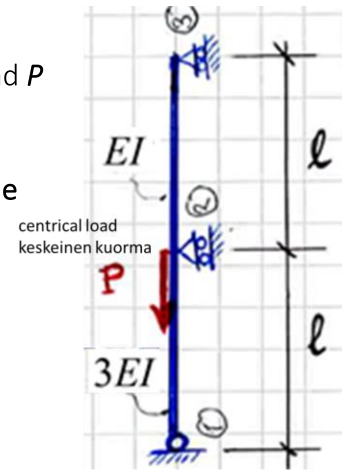
4. Buckling of continuous beam-columns

A continuous column is loaded at mid support 2 by a concentrated centrical load *P* (keskeinen kuorma, zero eccentricity)

Use Slope-Deflection Method and 1) derive the explicit expression, in terms of Berry's stability functions, of the needed **criticality condition for determining the critical buckling load** *P* [3 p].

2) **solve numerically for the value of the buckling load** *P* [1p].

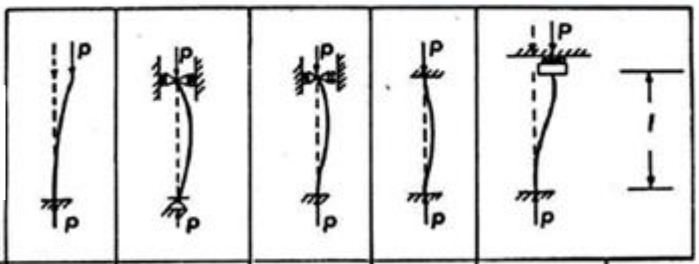
3) Give a **bracket** for the value of **buckling load** using cleverly the Euler's basic cases (see tables in the formulary) [1p].



The slope-deflection method:

Euler's basic buckling cases Eulerin perusnurjahdus

$$P_{cr} = \mu \frac{\pi^2 EI}{\ell^2}$$



Tapaus	1	2	3	4	5
μ	0.25	1	2.046	4	1

$$M_{ij} = A_{ij}\phi_{ij} + B_{ij}\phi_{ji} - C_{ij}\psi_{ij} + \bar{M}_{ij} \qquad M_{ij}^0 = A_{ij}^0\varphi_{ij} - C_{ij}^0\psi_{ij} + \bar{M}_{ij}^0$$

Beam-column with constant flexural rigidity:

$$A_{ij} = A_{ji} = \frac{2\psi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}, \qquad B_{ij} = B_{ji} = \frac{\phi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}$$

$$C_{ij} = A_{ij} + B_{ij} \qquad A_{ij}^0 = C_{ij}^0 = \frac{1}{\psi(kL)}\frac{3EI}{L}, \qquad kL \equiv L\sqrt{\frac{P}{EI}}$$

Berry's functions:

Olkoon $\lambda \equiv kL$, $\qquad \lambda \equiv kL$

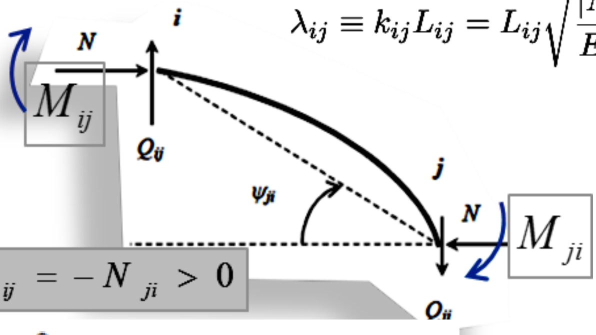
Puristettu sauva:

Compression: $\phi(\lambda) = \frac{6}{\lambda}\left(\frac{1}{\sin\lambda} - \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(\frac{1}{\lambda} - \frac{1}{\tan\lambda}\right)$, ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(\tan\frac{\lambda}{2} - \frac{\lambda}{2}\right)$,

Vedetty sauva:

Extension: $\phi(\lambda) = \frac{6}{\lambda}\left(-\frac{1}{\sinh\lambda} + \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(-\frac{1}{\lambda} + \frac{1}{\tanh\lambda}\right)$, ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(-\tanh\frac{\lambda}{2} + \frac{\lambda}{2}\right)$,

$$\lambda_{ij} \equiv k_{ij}L_{ij} = L_{ij}\sqrt{\frac{|N_{ij}|}{EI_{ij}}} \rightarrow \lambda \equiv kL = L\sqrt{\frac{|N|}{EI}}$$

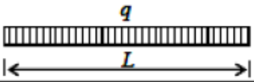


$$N \equiv -N_{ij} = -N_{ji} > 0$$

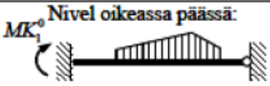
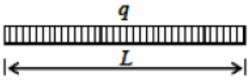
$$Q_{ij} = Q_{ij}^0 - (M_{ij} + M_{ji})/L - N\psi_{ij}$$

$$\overline{M}_{12} \equiv MK_1$$

$\overline{M}_{ij}, \overline{M}_{ji}$

N:o	Kuormitus	Kiinnitysmomentit: 
1		$MK_1 = -\frac{qL^2}{12}$, $MK_2 = \frac{qL^2}{12}$
2		

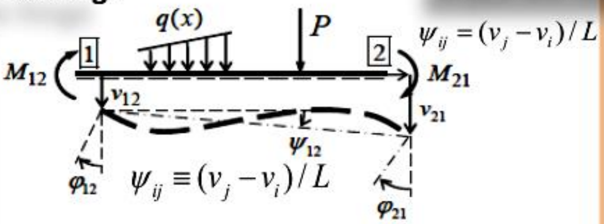
$\overline{M}_{ij}$ $\overline{M}_{ji}$

N:o	Kuormitus	Nivel oikeassa päässä: 	Nivel vasemmassa päässä: 
1		$MK_1^0 = -\frac{qL^2}{8}$	$MK_2^0 = \frac{qL^2}{8}$

The stiffness equation relating the end-moments to the end-displacements

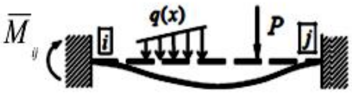
If you are using lecture's notations

No hinge



$$M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + \overline{M}_{ij}, \quad i \neq j$$

$$a_{ij} = \frac{4EI}{L}, \quad b_{ij} = \frac{2EI}{L}, \quad c_{ij} = \frac{6EI}{L} \quad (EI\text{-constant})$$



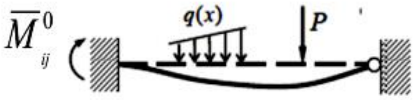
Fixed end-moment resulting from external mechanical loading, look from tables

One node is hinged

The is a superscript "0" means that the support at end j is hinged

$$M_{ij}^0 = a_{ij}^0\varphi_{ij} - c_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

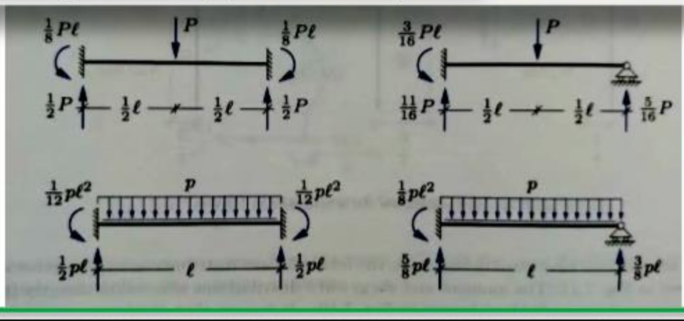
$$a_{12}^0 = c_{12}^0 = \frac{3EI}{L} \quad \psi_{ij} = (v_j - v_i)/L$$



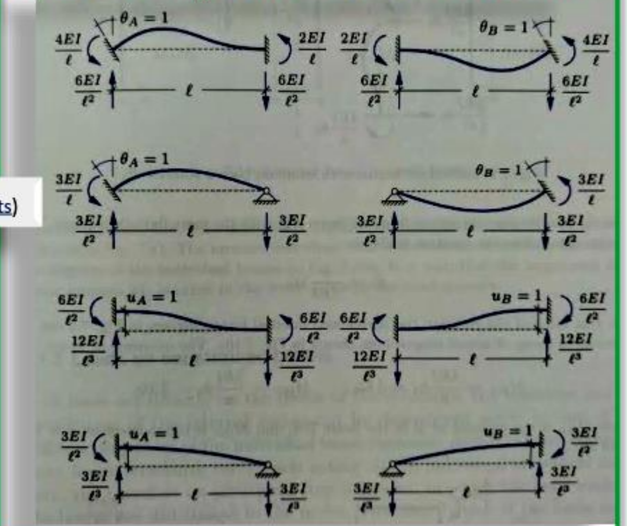
Fixed end-moment resulting from external mechanical loading, look from tables

If you are using Krenk's textbook notations

Bending Moments (pay attention to the sign convention to convert to Fixed-End-Moments)

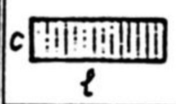
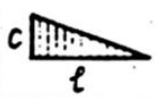
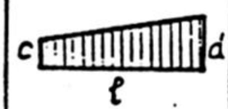
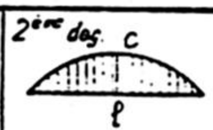
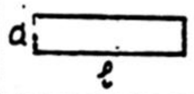
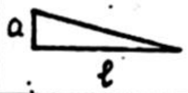
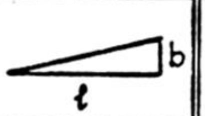
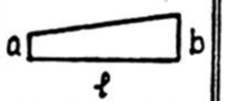
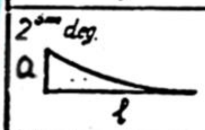
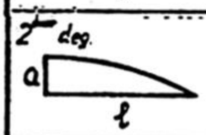


If you are using Krenk's textbook notations



Maxwell-Mohr integrals table

TABLEAU DES INTEGRALES $\int_0^\ell {}^iM\,{}^kM\,dx$

${}^kM$ \ ${}^iM$					
	$ac\ell$	$\frac{1}{2}ac\ell$	$\frac{1}{2}ad\ell$	$\frac{1}{2}a\ell(c+d)$	$\frac{2}{3}ac\ell$
	$\frac{1}{2}ac\ell$	$\frac{1}{3}ac\ell$	$\frac{1}{6}ad\ell$	$\frac{1}{6}a\ell(2c+d)$	$\frac{1}{3}ac\ell$
	$\frac{1}{2}bc\ell$	$\frac{1}{6}bc\ell$	$\frac{1}{3}bd\ell$	$\frac{1}{6}b\ell(c+2d)$	$\frac{1}{3}bc\ell$
	$\frac{1}{2}(a+b)c\ell$	$\frac{1}{6}(2a+b)c\ell$	$\frac{1}{6}(a+2b)d\ell$	$\frac{\ell}{6}[a(2c+d)+ b(c+2d)]$	$\frac{1}{3}(a+b)c\ell$
	$\frac{1}{3}ac\ell$	$\frac{1}{4}ac\ell$	$\frac{1}{12}ad\ell$	$\frac{1}{12}a\ell(3c+d)$	$\frac{1}{5}ac\ell$
	$\frac{2}{3}ac\ell$	$\frac{5}{12}ac\ell$	$\frac{1}{4}ad\ell$	$\frac{1}{12}a\ell(5c+3d)$	$\frac{7}{15}ac\ell$