Personal exam 9.4.2020, duration 4h + 35 min for doing the pdfs and uploading to MyCourses, can use books, computers, etc...

- Check that your result has correct physical dimensions (units)
- Exercises 1 and 3 are not difficult (start by them). The 2nd one, a simple but a bit laborious; use computer algebra, if you wish. The fourth is laborious.
- Solve all the four (4) exercises. Passing the exam needs $\geq$ 40% of 4x5=20 points = 8 points.

Exercise 1: Fundamentals of stability [5 points]

The perfectly vertical column AB is rigid and linked by a linear elastic spring to the vertical roller (Figure 1). The roller can freely move only vertically. The column rotation can be denoted by ϕ. There is two hinges at A and B.

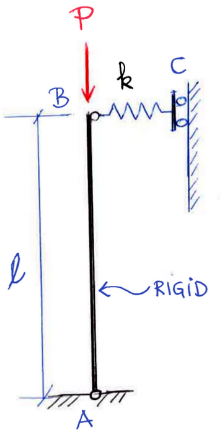


Figure 1: The spring coefficient k [N/m]. For the column, $EI \to \infty$ and $EA \to \infty$.

1. Determine the buckling load P_E
2. Determine and draw all equilibrium paths: the graphs $(P/P_E, \phi)$
3. Investigate the stability nature of all equilibrium paths (branches) and indicate clearly the stable and unstable branches

Exercise 2: Buckling of plates [5 points]

Consider the buckling of the thin linear elastic plate (figure 2). The plate is loaded by a constant in-plane distributed edge-load N_x^0.

1. Write down all the boundary conditions
2. Specify which boundary conditions are the *kinematic* ones
3. Determine an upper-bound estimate for the buckling edge load $N_{x,cr}^0$ of the plate in figure 2b). Use Rayleigh-Ritz method.

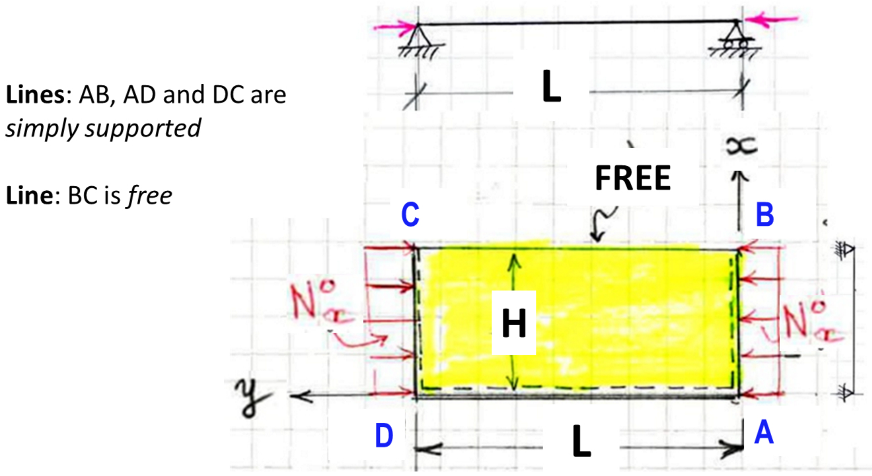


Figure 2: The plate is isotropic elastic with thickness h (thin). E, ν and bending rigidity D are given. [the rollers are only on the ends and direction of sides AB and DC to allow free expansion in $x-$ direction]

The energy functional: The increment of total potential energy[1]:

$$\Delta\Pi = \frac{1}{2}D\int_A \left[w_{,xx}^2 + w_{,yy}^2 + 2\nu w_{,xx}w_{,yy} + 2(1-\nu)w_{,xy}^2\right] \mathrm{d}A + \quad (1)$$

$$+\frac{1}{2}\int_A \left[N_{xx}^0 w_{,x}^2 + N_{yy}^0 w_{,y}^2 + 2N_{xy}^0 w_{,x}w_{,y}\right] \mathrm{d}A \quad (2)$$

[1]Notation: $f_{,\alpha} = \partial f/\partial\alpha$

Exercise 3: Flexural Buckling of a column [5 points]

Question: *Estimate the buckling load.*

Consider flexural buckling (in the plan of the paper) of the cantilever column with variable cross-section (Figure 3). The load P is centric (applied at the center of gravity of the cross section). The inertia moment of the cross-section

$$I(x) = I_0 \cdot [2 - (x/\ell)^2], \tag{3}$$

where I_0 (units $[\mathrm{m}^4]$) being constant. The material of the beam is homogeneous with elasticity modulus E.

Assume that the column works as a beam and not as a plate and use energy principles to estimate the buckling load P_{cr}.

Hints: what are the kinematic boundary conditions? Use the simplest approximation for the deflection $v(x)$. Clearly, the buckling load will be greater than for a column with constant cross-section having constant $EI = EI_0$.

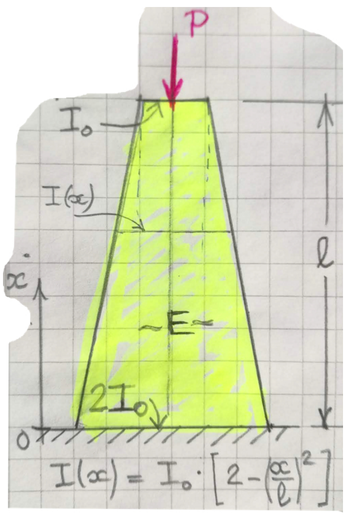


Figure 3: Buckling of a column with non-constant cross-section.

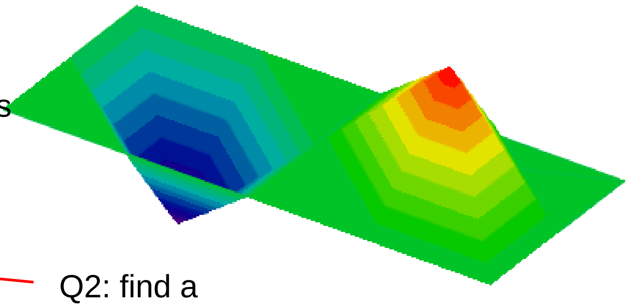
Exercise 4: (5 points)

This exercise is challenging and shows you that you can do real structural design using also science based theory. It also tests if the student can apply what he has studied to new situations.

In bridge construction several methods to increase the span length is utilized. Spans between 6 to 38 meters are categorized in short span bridges which I steel girder ranging less than 50 meters is very common. The cross-section of these girders consists of two flanges and a long thin web. The stiffeners are used to prevent shear buckling. As a structural designer, assume the plate between two flanges and two stiffeners has a shape function of:

$$w(x,y) = w_0 . \sin\left(\frac{\pi x}{a}\right) . \sin\left(\frac{\pi y}{a}\right) . \sin\left(\frac{\pi x}{a} - \frac{\pi y}{a}\right)$$

Where: a is the longitudinal spacing of the bay, b is the distance between flanges and the shape function satisfies the boundary condition as shown. Derive to the shear buckling as an equation of D, a, and b:

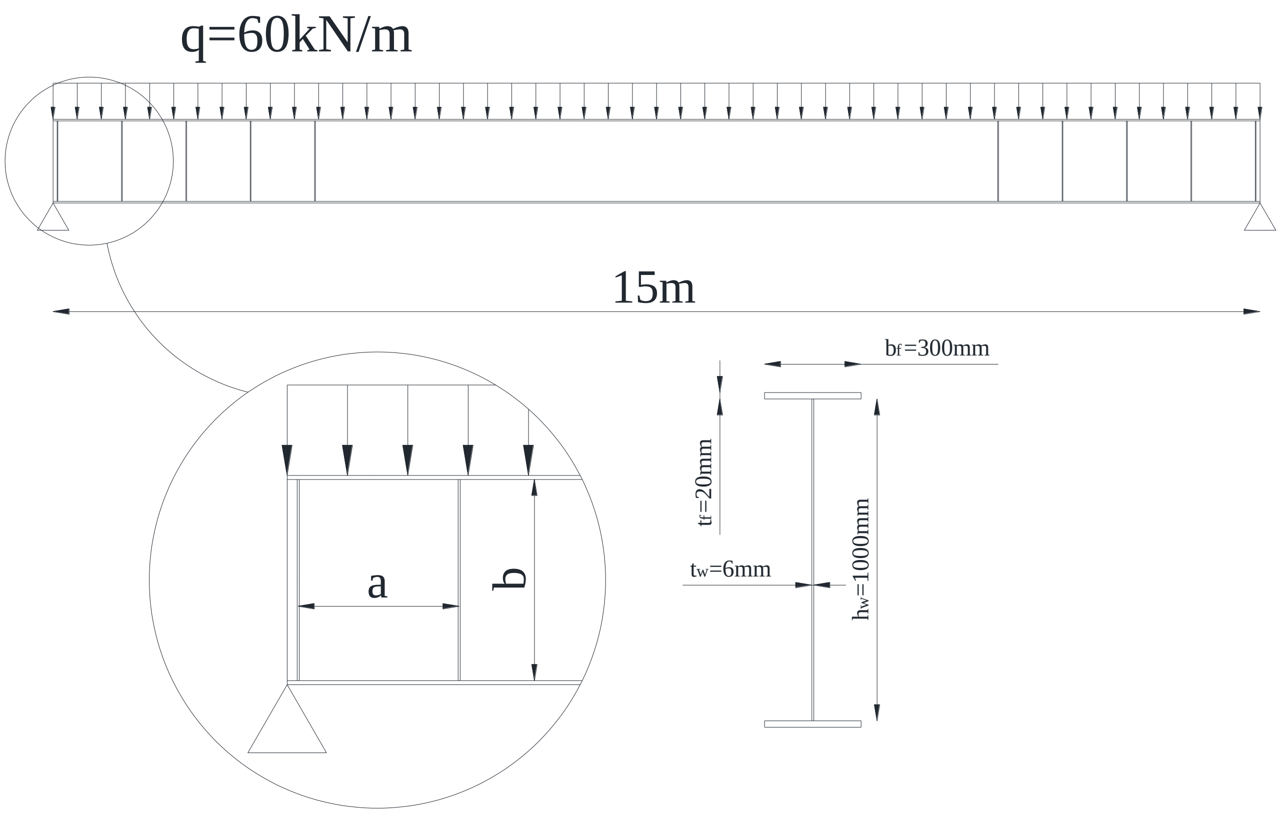


$$N_{xy_{cr}} = \frac{4\pi^2 D}{a^3 b^3}(a^4 + a^2 b^2 + b^4)$$

Q1: derive this equation

If a plate girder is used for a span of 15 meters under the ultimate distributed load shown in the figure below, define the spacing of the stiffeners in such a way that the shear buckling capacity of the girder in each bay at supports is 5 times of the shear buckling of the plate, find a. Modulus of elasticity and Poisson's ratio are 200GPa and 0.3 respectively.

Q2: find a



N.B. : Do not consider lateral torsional buckling of the beam in the exam. The beam is correctly restrained against such torsional loss of stability.