

**Aalto University**

Björn Ivarsson, 050-4067 832

**Exam, Monday February 22nd 2021, 13:00 - 17:00**

Complex Analysis, MS-C1300

Motivate your answers. Only giving answers gives no points.

See exam instructions here:

[mycourses.aalto.fi/course/view.php?id=29625&section=7](https://mycourses.aalto.fi/course/view.php?id=29625&section=7)

- (1) (a) Let  $z = x + iy \in \mathbb{C}$ . Determine  $v(x, y)$  so that

$$f(z) = x^2 - y^2 + x + iv(x, y)$$

is analytic. Justify your answer. (3p)

- (b) Let  $z = x + iy \in \mathbb{C}$  and  $U = \{z \in \mathbb{C}; x > 0\}$ . Is

$$f(z) = \frac{1}{2} \ln(x^2 + y^2) + i \arctan\left(\frac{y}{\sqrt{x^2 + y^2}}\right)$$

analytic in  $U$ ? Justify your answer. (3p)

- (2) Let  $a \neq 0$  be a complex number.

- (a) Find the Taylor series of

$$f(z) = \frac{z}{z + a}$$

around  $z_0 = 0$ . Determine the radius of convergence  $\rho$  for the series. (2p)

- (b) Find the Laurent series of

$$f(z) = \frac{z}{z + a}$$

in  $\{z \in \mathbb{C}; |a| < |z| < \infty\}$ . (2p)

- (c) Find the Taylor series of

$$f(z) = \frac{z}{z - 2a}$$

around  $z_0 = a$ . Determine the radius of convergence  $\rho$  for the series. (2p)

- (3) Let  $a > 0$  be a real number. Calculate

$$\int_0^\infty \frac{\cos x}{x^2 + a^2} dx.$$

(6p)

(4) Let  $a > 0$  be a real number. Determine all poles  $z_k$  of

$$f(z) = \frac{z}{e^{az} - 1}$$

and also determine the order of the poles. Calculate the residues  $\text{Res}(z_k, f)$ . (6p)

**Good luck!**