Aalto University
School of Science

PHYS-E0420 Many-Body Quantum Mechanics
Midterm Exam 24.02.2020

General advice: You are allowed to use material (lecture noes, internet, books...), but collaboration is not permitted.

Do not panic if you feel you cannot complete all tasks perfectly in the given time. The test is on purpose made quite broad; the results will be scaled in the end so that you will receive in the average similar grades the students of previous years got from usual exams. RETURN YOUR SOLUTIONS AS A SINGLE CLEARLY READABLE PDF-FILE.

1. Describe all the approximations done in deriving the Fermi's golden rule, and define its regimes of validity. Find from the internet three examples (in different systems, different contexts) of the usage of the Fermi's golden rule and describe them briefly.

2. Show that for two particle operators

$$F = \frac{1}{2}\sum_{\alpha\neq\beta} f^{(2)}(\mathbf{x}_\alpha, \mathbf{x}_\beta) \tag{1}$$

one obtains

$$F = \frac{1}{2}\sum_{i,j,k,m} \langle i,j|f^{(2)}|k,m\rangle a_i^\dagger a_j^\dagger a_m a_k, \tag{2}$$

where

$$\langle i,j|f^{(2)}|k,m\rangle = \int d\mathbf{x} \int d\mathbf{y} \varphi_i^*(\mathbf{x})\varphi_j^*(\mathbf{y}) f^{(2)}(\mathbf{x},\mathbf{y})\varphi_k(\mathbf{x})\varphi_m(\mathbf{y}). \tag{3}$$

3. Show that the potential energy term can be expressed with field operators as

$$H_{pot} = \int d^3x U(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \tag{4}$$

4. Write a mini-essay (about 1-2 pages) on the topic "Quantum states of the electromagnetic field". You may use selected equations, but lengthy derivations are not recommended. Instead, try to explain the essential ideas of the different states by words. Pictures and illustrations are welcome. Find from the internet three examples of the usage of these quantum states in some other context than electromagnetic field.

5. In general, the oscillator strength can be either positive or negative, depending on the sign of $\omega_{n'} - \omega_n$ that appears in its definition (in other words, whether state n has higher or lower energy than n').

   Now consider the derivation of the Thomas-Reiche-Kuhn sum-rule presented in the lectures. One might claim that there is a contradiction if one takes n to be the highest energy state, since then all $f_{nn'}$ will be negative and cannot add up to one. However, there is a problem in this claim, and actually no contradiction exists. Why not? What is the problem with the claim?

   Hint: consider the step going from $\langle n|\hat{p}_x|n'\rangle$ to $\langle n|\hat{x}|n'\rangle$ and what is it based on.