MS-E2148 Dynamic optimization Harri Ehtamo and Anton von Schantz

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-Online exam
-Starting: Thursday 25.2.2021 at 15:00
-Ending: Friday 26.2.2021 at 15:00
-Solutions should be clearly handwritten or LaTeX-compiled
-Upload solutions as pdf, jpg or png files

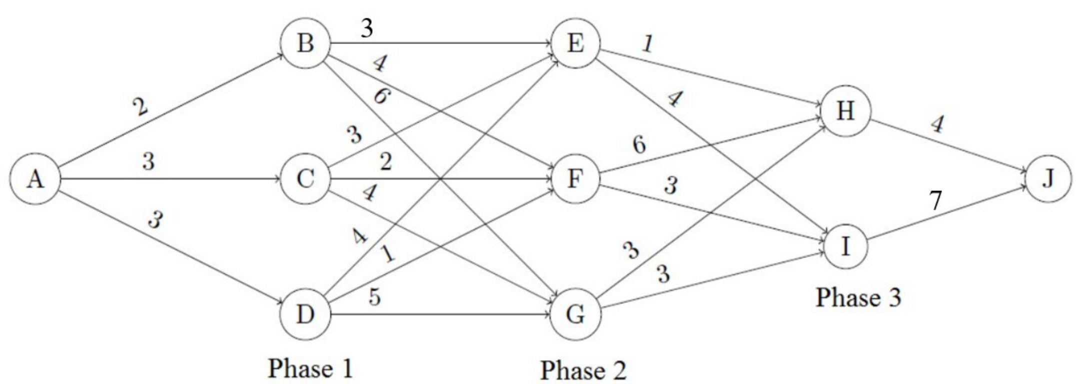
1. Explain shortly.
   a) What does the variation of a functional mean.
   b) What are the corner point conditions.
   c) What are the transversality conditions.
   d) What is the Pontryagin's minimum principle.
   e,f) What is the Euler-Lagrange equation and how is it derived.

2. Find the extremal of the functional

$$J(x) = \int_0^1 [\dot{x}(t)^2/2 + x(t)(\dot{x}(t)+1)]dt,$$

   when $x(0) = 0$ and $x(1)$ is free.

3. The graph below depicts a flexible production line. The initial product A has to go through three phases to become the final product J. When the product goes from one node to another, the value, or the profit the product will give, is increased. The profit from each node to another is given next to the corresponding arc.



   Calculate the optimal cost-to-go function for each node and determine the steps that have to be taken for to *maximize* the profit.

4. Consider a thief gets into a home to rob and he carries a knapsack. There are a fixed number n of items in the home each i with its own weight w_i and value v_i. The thief has an old knapsack which has limited capacity. At most it can carry the weight W. The thief's problem is to choose which items to pack so that he maximizes the total value of the stolen items.

   Derive the DP algorithm that can be used to solve the thief's problem.

5. Your friend Robert Goddard is building the first liquid-fueled rocket in 1926 (see Fig. 1). Your task is to help Goddard in getting the rocket as high as possible in the sky. You may assume that gravity is constant and the drag (or air resistance) depends on the speed v and altitude h of the rocket. The drag is given by $D(v, h) = av^2e^{-bh}$, where $a > 0$ and $b > 0$ are constants. The thrust (or the force describing the acceleration of the rocket) F is restricted between 0 and F_{max}, and it holds for the change of the rocket's mass is given by $\dot{m} = -F/c$, where $c > 0$ is a constant.

   Formulate the problem mathematically and solve the optimal control as a function of the costate variables. Describe the solution.

Figure 1: Goddard and the rocket.

APPENDIX:

HJB: $0 = J_t + \min_{u(t)} \left\{ g + J_x^T f \right\}$

E-L: $0 = g_x - \frac{d}{dt}(g_{\dot{x}})$

Hamiltonian: $H = g + p^T(t) f(x(t), u(t), t)$

costate: $\dot{p}(t) = -\frac{\partial H}{\partial x}$

free final state: $0 = g_{\dot{x}}$ or $h_x - p = 0$

free final time: $0 = g - g_{\dot{x}}\dot{x}$ or $H + h_t = 0$

free final state and time: $g_x = g_{\dot{x}} = 0$ or $h_x - p = 0 = H + h_t$

goal: $0 = g + \left[\frac{\partial g}{\partial \dot{x}}\right]^T \left[\frac{d\theta}{dt} - \dot{x}\right]$ or $H + h_t + (h_x - p)^T \frac{d\theta}{dt} = 0$

W-E: $g_{\dot{x}}$ and $g - g_{\dot{x}}\dot{x}$ continuous