Process control and automation (CHEM-C2140)
Exam, part 2: mathematical problems

April 12, 2021

Q5) A Flow meter can detect flows up to 400 l/min. A flow of 400 l/min produces a sensor output of 20 mA while flow of 0 l/min produces a sensor output of 4 mA.

(a) What are the transmitter gain, zero and span?

(b) Develop a linear expression for the flow meter.

(4p)

Q6) The water level inside a water tower (see Fig 1) is controlled by manipulating the operating power of a pump, located at the ground level. The tower has a height of H, and its tank cross-sectional are of A. The pumping power and the mass flow rate into the tank, w_{in}, have the following relation:

$$P = w_{\text{in}} g H$$

where q is the gravitational acceleration. The tower has an outlet stream (the mass flow rate of which is w_{out}) that leads to the regional water supply network. Derive a physics-based model for the water level h in the tank of the tower as a function of the pumping power P, the outlet mass flow rate w_{out}, the height of the tower H, the density of the water ρ, and the gravitational acceleration g.

Follow the steps of the systematic model development. State all assumptions that are needed, and pay special attention to the classification of inputs.

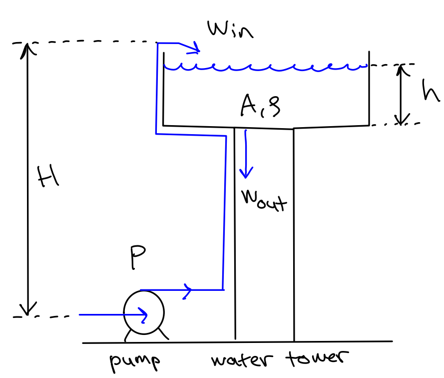

Figure 1: Jacketed continuous process reactor.

(8p)

Q7) Let us consider the liquid tank shown in Fig. 2. The physics-based model for liquid level $h(t)$ in the tank is

$$\frac{dh(t)}{dt} = \frac{1}{A}\Big(q_{\text{in}} - q_{\text{out}}\Big),$$

where A is the cross-sectional area of the tank. The volumetric flow rate into the tank is $q_{\text{in}} = cu(t)$, where c is the valve coefficient and $u(t) \in [0, 1]$ is the position of the valve. The actuator of the valve reacts immediately to its control signal, and the length of the inlet pipe is negligible for the process dynamic. The volumetric flow rate at the outlet, q_{out}, is linearly proportional to the liquid level h in the tank, $q_{\text{out}} = rh(t)$, where r is a constant. The density of the liquid is ρ. The model is assume to be a precise representation of the liquid level $h(t)$ in the tank.

Considering the valve position $u(t)$ as the input variable and the tank level $h(t)$ as the output variable, identify the process parameters (i.e., the process gain K_{p}, the process time coefficient τ_{p}, and the process dead time θ_{p}) for the liquid tank without performing a step test.

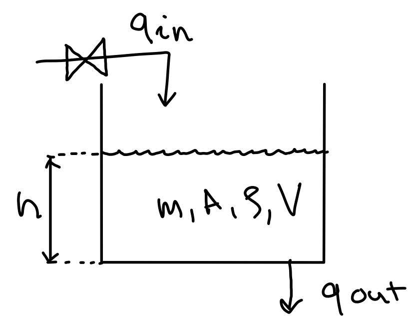

Figure 2: A liquid tank with a hole.

(5p)

Q8) A liquid vessel is heated by a heating rate $Q(t)$ (see Fig. 3). During the heating process, heat is also transferred from the vessel to the surrounding air, characterized by the heat transfer coefficient h (unit: W/(m^2 °C)). The temperature distribution inside the vessel is assumed to be uniform. A physics-based model for the liquid temperature $T(t)$ is

$$mc\frac{dT(t)}{dt} = Q(t) + hA(T_{\text{amb}} - T(t)),$$

where $m = 1000$ kg is the mass of water in the vessel, $c = 4$ kJ/(kg °C) is the specific heat capacity of water, $h = 100$ W/(m^2 °C) is the heat transfer coefficient, $A = 5$ m^2 is the surface area of the vessel, $T_{\text{amb}} = 10$ °C is the ambient temperature of the surrounding air. The initial condition for the liquid temperature is $T(0) = 30$ °C. The heater is switched on to rate $Q_{\text{nom}} = 25$ kW at time $t = 0$, and kept constant after this.

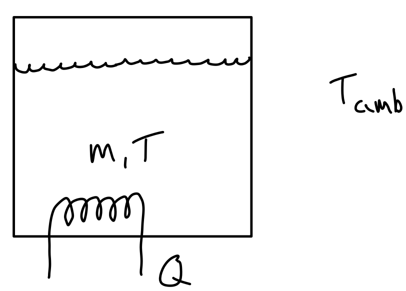

Figure 3: A liquid vessel with a heater.

a) Transform the heating rate $Q(t)$ into the Laplace domain (i.e., determine $Q(s)$). (2p)

b) Transform the given physics-based model into the Laplace domain. (2p)

c) Solve the liquid temperature $T(s)$ in the Laplace domain and transform it back to time domain, in order to obtain an analytical expression for $T(t)$.[1] (4p)

[1]You may plot the expression using the attached Jupyter Notebook Q8_verification.ipynb to verify its steady-state value after one day of heating. The notebook is only for verification. It is not meant to be submitted.