First course in probability and statistics
MS-A0501, MS-A0502, MS-A0503, MS-A0504 — Exam
Department of Mathematics and Systems Analysis — 13.4.2021
Aalto University — J Kohonen, G Metsalo

Instructions: Write briefly and clearly, but give reasons for your answers. A number only as an answer does not yield points. The exam has 4 problems, each worth 0–6 points.

Write your answers in clear **handwriting**, either on paper or a tablet computer, and send them in PDF form to the return box on the exam page. Make sure your answers contain: course code, last name, first name, student number and date. At the end of the exam sheet there is a table for the CDF of the standard normal distribution.

P1 The PIN code of a debit card contains four digits, each of which is taken independently at random from the discrete uniform distribution over the set $\{0, 1, \ldots, 9\}$.

(a) What is the probability that all digits are even? **(1p)**

(b) What is the probability that all digits are the same? **(1p)**

(c) What is the probability that the code does not contain any digit twice? **(1p)**

(d) What is the probability that two of the digits are even and two are odd? **(2p)**

(e) What is the expected number of even digits? **(1p)**

P2 At each point in time $t \in \mathbb{R}$, an alternating current has voltage $u(t) = \sin(t)$ and power $p(t) = u(t)^2$. We pick a random point in time T from the continuous uniform distribution over the interval $[0, 2\pi]$. We define random variables $U = u(T)$ and $P = p(T)$. Calculate:

(a) $\mathbb{E}(T)$ **(1p)**

(b) $\mathbb{E}(U)$ **(2p)**

(c) $\mathbb{E}(P)$ **(3p)**

Useful formulas from trigonometry:

$$\int \sin(x)\, dx = -\cos(x) + C$$
$$\int \cos(x)\, dx = \sin(x) + C$$
$$(\sin(x))^2 = \big(1 - \cos(2x)\big)/2$$

P3 Let s be the **last digit of your student number** (one of $0, \ldots, 9$, ignore possible letters). Write the value of s clearly in your answer and use this value in your calculations.

Insects hit the windscreen of a car randomly. The first one hits X_1 minutes after start, and then the intervals are $X_2, X_3, \ldots$ minutes. Each X_i is independent and exponentially distributed with rate parameter $\lambda = 3 + (s/10)$ insects per minute, that is, with density function

$$f(t) = \lambda e^{-\lambda t},$$

when $t > 0$. It is known that $\mathbb{E}(X_i) = \mathrm{SD}(X_i) = 1/\lambda$.

Mr. K drives until 50 insects have hit the windscreen. This takes time $S = X_1 + \ldots + X_{50}$.

(a) Calculate $\mathbb{E}(S)$ ja $\mathrm{SD}(S)$ with three decimals. **(2p)**

(b) Calculate, using the normal approximation, the approximate probability that the total time exceeds 17 minutes. **(2p)**

(c) Calculate the probability that at least one of the intervals $X_1, X_2, \ldots, X_{50}$ exceeds 1.2 minutes. Do not use normal approximation. **(2p)**

P4 In three languages (English, Swedish, Finnish), the relative frequencies of letters are as in the table on the following page. We assume that in a text, each letter is taken independently at random according to the frequencies of the language, so for example in Finnish text, each letter is A with probability 0.121.

On a certain channel, 80% of messages are English, 10% are Swedish, and 10% are Finnish. One message was intercepted from the channel at random, and its text is: ”SALAKKA”. Calculate the posterior distribution of its language, that is, calculate the probabilities that it is English, that it is Swedish, and that it is Finnish.

Table for problem P4.

letter	English	Swedish	Finnish
A	0.081	0.094	0.121
B	0.015	0.015	0.003
C	0.028	0.015	0.003
D	0.042	0.048	0.010
E	0.127	0.101	0.080
F	0.021	0.020	0.002
G	0.020	0.030	0.004
H	0.061	0.021	0.019
I	0.070	0.058	0.108
J	0.002	0.006	0.020
K	0.008	0.031	0.050
L	0.040	0.053	0.058
M	0.024	0.035	0.032
N	0.067	0.085	0.088
O	0.075	0.045	0.056
P	0.019	0.018	0.018
Q	0.001	0.000	0.000
R	0.060	0.084	0.029
S	0.063	0.066	0.079
T	0.091	0.077	0.088
U	0.028	0.019	0.050
V	0.010	0.024	0.023
W	0.024	0.001	0.001
X	0.002	0.002	0.000
Y	0.020	0.007	0.017
Z	0.001	0.001	0.001
Å	0.000	0.013	0.000
Ä	0.000	0.018	0.036
Ö	0.000	0.013	0.004

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Normal distribution table

The table below contains numerical values of the standard normal cumulative distribution function

$$\Phi(x) = F_Z(x) \;=\; \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999