## FINAL EXAM, FOUNDATIONS OF DISCRETE MATHEMATICS, MS-A0402

- Time: 9:00-13:00
- You may use any material you wish, but you must not communicate with anybody during the exam.
- Your solutions should be hand-written on paper or tablet.
- Your name and student number must be visible on each page.
- Each problem is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points.


## Problem 1

Are the following statements tautologies or not? (1p/part)
a) $(P \vee Q) \vee(\neg P \wedge \neg Q)$.
b) $(P \rightarrow Q) \vee(Q \rightarrow P)$.
c) $((P \rightarrow Q) \wedge(Q \rightarrow R)) \rightarrow(P \rightarrow R)$.
d) $((P \wedge Q) \vee R) \leftrightarrow((P \wedge R) \vee(Q \wedge R))$.

## Problem 2

Let $|A|=n$. Find a formula (in terms of $n$ ) for the number of functions $f: A \rightarrow$ $P(A)$ such that $\forall x \in A: x \in f(x)$.

## Problem 3

Consider the relation $\preceq$ on $\mathbb{Z}$ given by $x \preceq y$ if there exist $m \in \mathbb{N}$ and an odd integer $k$ such that $k x=2^{m} y$.
a) Prove that $\preceq$ is an order relation on $\mathbb{Z}$. ( 2 p )
b) Draw the Hasse diagram of $\preceq$ on the set $\{0,1,2, \ldots, 10\}$ (1p)
c) Give an example of a linear extension of $\preceq$ on the set $\{0,1,2, \ldots, 10\}$. (1p)

Problem 4
In the weighted graph below, the red edges form a minimal spanning tree. What can you say about the unknown weight $x \in \mathbb{R}$ ?


Problem 5
We know that

$$
2^{11}=2048
$$

and that

$$
2021=43 \cdot 47
$$

Using these facts, find a number $k$ such that

$$
27^{k} \equiv 2 \quad \bmod 2021
$$

