FINAL EXAM, FOUNDATIONS OF DISCRETE MATHEMATICS, MS-A0402

- Time: 9:00-13:00
- You may use any material you wish, but you must not communicate with anybody during the exam.
- Your solutions should be hand-written on paper or tablet.
- Your name and student number must be visible on each page.
- Each problem is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points.

Problem 1

Are the following statements tautologies or not? (1p/part)

- a) $(P \lor Q) \lor (\neg P \land \neg Q).$
- b) $(P \to Q) \lor (Q \to P)$.
- c) $((P \to Q) \land (Q \to R)) \to (P \to R).$

d) $((P \land Q) \lor R) \leftrightarrow ((P \land R) \lor (Q \land R)).$

Problem 2

Let |A| = n. Find a formula (in terms of n) for the number of functions $f : A \to P(A)$ such that $\forall x \in A : x \in f(x)$.

Problem 3

Consider the relation \leq on \mathbb{Z} given by $x \leq y$ if there exist $m \in \mathbb{N}$ and an odd integer k such that $kx = 2^m y$.

- a) Prove that \leq is an order relation on \mathbb{Z} . (2p)
- b) Draw the Hasse diagram of \leq on the set $\{0, 1, 2, \dots, 10\}$ (1p)
- c) Give an example of a linear extension of \leq on the set $\{0, 1, 2, \dots, 10\}$. (1p)

Problem 4

In the weighted graph below, the red edges form a minimal spanning tree. What can you say about the unknown weight $x \in \mathbb{R}$?



Problem 5

We know that

$$2^{11} = 2048$$

and that

$$2021 = 43 \cdot 47.$$

Using these facts, find a number k such that

$$27^k \equiv 2 \mod 2021.$$