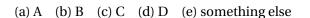
ELEC–C9430 Electromagnetism — Spring 2021 / IV

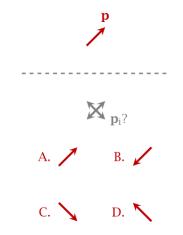
THE FIRST PROBLEM

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This problem has six multiple-choice questions. Choose, for each question, **one** and **only one** of the answers. No need to justify your answer.

- 1. When is the line integral $\oint_C \mathbf{F}(\mathbf{R}) \cdot d\ell$ equal to zero (for any closed contour *C*)? (Here $d\ell$ is the differential vectorial line element along the contour.)
 - (a) Always.
 - (b) If and only if function **F** is divergenceless.
 - (c) If and only if function **F** is curl-free.
 - (d) Provided another condition is valid.
 - (e) Never.
- 2. A static electric dipole **p** is located above a perfectly conducting plane. To solve the the field by the image principle, we need the image dipole \mathbf{p}_i at the mirror image point. The image dipole has the same amplitude as the original dipole: $|\mathbf{p}_i| = |\mathbf{p}|$. What is the direction of the image dipole?



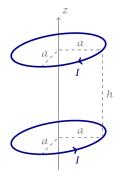


3. Claim 1: If at a given point in space the scalar potential V = 0, then also the static electric field at the same point $\mathbf{E} = 0$.

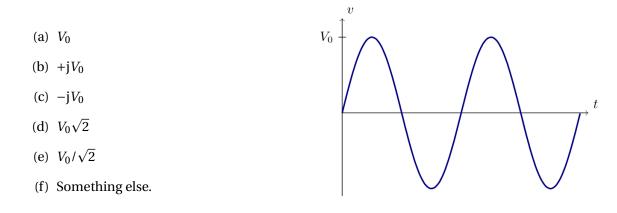
Claim 2: If at a given point in space the static electric field $\mathbf{E} = 0$, then also the scalar potential at the same point V = 0.

Which of the following holds?

- (a) only Claim 1
- (b) only Claim 2
- (c) both Claims
- (d) neither Claim 1 nor Claim 2.
- 4. Two circular current loops are located as in the picture, with the currents directed according to the arrows. They experience towards each other
 - (a) an attracting force
 - (b) a repelling force
 - (c) no force at all.



5. What is the complex scalar that corresponds to the time-dependent voltage on the picture v(t)?



- 6. The electric field function of a plane wave at z = 0 is $\mathbf{E} = E_0(\mathbf{a}_x + \mathbf{j}\mathbf{a}_y)$. What is the polarization of this plane wave?
 - (a) Linear
 - (b) Elliptic, right-handed
 - (c) Elliptic, left-handed
 - (d) Circular, right-handed
 - (e) Circular, left-handed
 - (f) It cannot be determined without further information.

THE SECOND PROBLEM

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- 2. Answer concisely (shortly but informatively) the following in words.

(In case you need, you can use equations in addition.)

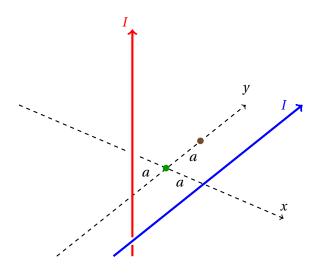
- (a) Explain the displacement current.
- (b) Is a superposition (sum) of two plane waves a plane wave? Give a reasoning for your answer.
- (c) A given electromagnetic wave propagates in a non-magnetic (μ = μ₀) lossy material, which has permittivity ε and conductivity σ. It is attenuated 10 dB/m at frequency ω.
 Suppose we increase the frequency by 10 %, in other words to 1.1ω. What happens to the attenuation? Is it stronger (more attenuation), weaker (less attenuation), the same, or can this be even determined without further information? Justify shortly your answer.

THE THIRD PROBLEM

3. Two long straight current wires are perpendicular to each other as in the picture. Both carry a steady current (DC current) of magnitude *I*.

The current in blue flows parallel to the *y* axis, in the *xy*-plane through the point x = a, z = 0. The current in red flows parallel to the *z* axis, in the *yz*-plane through the point x = 0, y = -a.

- (a) Determine the magnetic field vector in point (x, y, z) = (0, 0, 0) (the green point in the picture).
- (b) Determine the magnetic field vector in point (x, y, z) = (0, a, 0) (the brown point in the picture).
- (c) On which point is the magnetic field larger? How many percent larger?



THE FOURTH PROBLEM

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- 4. A plane wave propagates in free space (ε_0 , μ_0), in a region where there are no sources ($\rho_v = 0$, $\mathbf{J} = 0$). Its time-harmonic complex electric field function reads

$$\mathbf{E}(\mathbf{R}) = (\mathbf{j}\mathbf{a}_v + \mathbf{a}_z) B e^{+\mathbf{j}kx}$$

where $k = \omega \sqrt{\mu_0 \varepsilon_0}$, and *B* is a (possibly complex) scalar constant with unit V/m.

- (a) Determine the complex magnetic field function $H(\mathbf{R})$ for this plane wave.
- (b) Compute from your electric and magnetic fields the average power density of this plane wave. What is the direction of power flow? Does it make sense compared with the phase function of the electric field vector?

(Hint: use the complex Poynting vector in Equation (7-79) of the textbook.)

(c) How many watts per square meter does this plane wave carry in the case when B = (2 + j3) V/m?