RATKAISUT 16.4.2021

1. $\bar{r}(t) = (3t^2, 4t^3, 3t^4)$, $-1 \le t \le 2$, $\bar{r}(-1) = (3,-4,3)$, $\bar{r}(2) = (12,32,48)$

$$\Rightarrow \bar{r}\,'(t) = 6t\,\bar{i} + 12t^2\,\bar{j} + 12t^3\,\bar{k}$$

$$\bar{F}(\bar{r}(t))\cdot\bar{r}\,'(t) = 3t^4\cdot 6t + 4t^3\cdot 12t^2 + 3t^2\cdot 12t^3 = 102t^5$$

a) $$\int_C \bar{F}\cdot d\bar{r} = \int_{-1}^{2} 102t^5\,dt = \frac{102}{6}\Big/_{-1}^{2} t^6 = 17(64-1) = \underline{\underline{1071}}$$

b) $\bar{F}$ TOTEUTTAA EHDOT $\dfrac{\partial F_i}{\partial x_j} = \dfrac{\partial F_j}{\partial x_i}$, JOTEN SILLÄ ON POTENTIAALI $(D = \mathbb{R}^3)$

VAATIMUS: $\varphi_x = z$, $\varphi_y = y$, $\varphi_z = x \Rightarrow \varphi = xz + \frac{1}{2}y^2 (+C)$

$$\Rightarrow \int_C \bar{F}\cdot d\bar{r} = \varphi(12,32,48) - \varphi(3,-4,3) = \underline{\underline{1071}}$$

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2. $\bar{F}(x,y) = yx^2\,\bar{i} + xy\,\bar{j} \Rightarrow \dfrac{\partial F_2}{\partial x} - \dfrac{\partial F_1}{\partial y} = y - x^2$

$$\oint_{\partial D} y x^2\,dx + xy\,dy = \iint_D (y - x^2)\,dA = \int_0^{2\pi}\int_1^3 (r\sin\theta - r^2\cos^2\theta)\,r\,dr\,d\theta$$

$$= 0 - \int_0^{2\pi}\cos^2\theta\,d\theta \int_1^3 r^3\,dr = -\pi\cdot\frac{1}{4}\Big/_1^3 r^4 = -\frac{\pi}{4}(81-1) = \underline{\underline{-20\pi}}$$

SYMMETRIAN NOJALLA $\iint_D y\,dA = 0$ SUORAAN!

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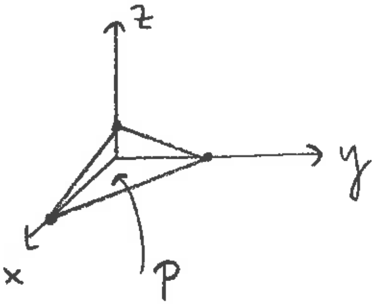
3. $$\nabla(\bar{u}\cdot\bar{r}) = \nabla(ax+by+cz) = a\bar{i} + b\bar{j} + c\bar{k} = \underline{\underline{\bar{u}}}$$

$$\bar{u}\times\bar{r} = (bz-cy)\bar{i} + (cx-az)\bar{j} + (ay-bx)\bar{k} \Rightarrow \nabla\cdot(\bar{u}\times\bar{r}) = 0+0+0 = \underline{\underline{0}}$$

$$(\bar{r}\cdot\nabla)\bar{r} = (x\partial_x + y\partial_y + z\partial_z)(x\bar{i} + y\bar{j} + z\bar{k}) = x\bar{i} + y\bar{j} + z\bar{k} = \underline{\underline{\bar{r}}}$$

TOISESSA VOI MYÖS KÄYTTÄÄ $\nabla\cdot(\bar{F}\times\bar{G}) = (\nabla\times\bar{F})\cdot\bar{G} - \bar{F}\cdot(\nabla\times\bar{G})$

4. TASON YHTÄLÖ: $x/3 + y/2 + z = 1$, YLÄTAHKO = P



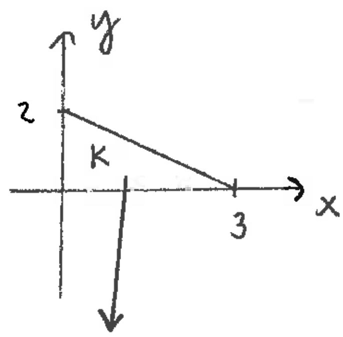
NORMAALI: $\bar{N} = \frac{1}{3}\bar{i} + \frac{1}{2}\bar{j} + \bar{k}$

b)
$$\left.\begin{array}{llll} x=0 & \Rightarrow \bar{m} = -\bar{i} & \Rightarrow \bar{F}\cdot\bar{m} & = -x = 0 \\ y=0 & \Rightarrow \bar{m} = -\bar{j} & \Rightarrow \bar{F}\cdot\bar{m} & = -y = 0 \\ z=0 & \Rightarrow \bar{m} = -\bar{k} & \Rightarrow \bar{F}\cdot\bar{m} & = -z = 0 \end{array}\right\} \Rightarrow \text{VUOT} = 0$$

c) SAMA $\bar{N}$ TULEE MYÖS YHTÄLÖSTÄ $z = 1 - \frac{1}{2}y - \frac{1}{3}x$, JOTEN

$\bar{m}\, dS = \bar{N}\, dA$ (VRT. MATERIAALIT)

$$\Rightarrow \iint_P \bar{F}\cdot\bar{m}\, dS = \iint_K 1\, dA = \frac{1}{2}\cdot 2\cdot 3 = \underline{\underline{3}}$$



$\bar{F}\cdot\bar{N} = \frac{x}{3} + \frac{y}{2} + z = 1$ PINNALLA P

TAI (ii): $\nabla\cdot\bar{F} = 1 + 1 + 1 = 3$

$$\iint_P \bar{F}\cdot\bar{m}\, dS \overset{b)}{=} \iint_{\partial T} \bar{F}\cdot\bar{m}\, dS \overset{\text{GAUSS}}{=} \iiint_T \nabla\cdot\bar{F}\, dV = 3\cdot\frac{1}{3}\cdot\underbrace{1}_{h \text{ ELI KORKEUS}}\cdot\overbrace{\frac{1}{2}\cdot 2\cdot 3}^{A} = \underline{\underline{3}}$$

5.
$$\iiint_K z\, dV = \int_0^{2\pi}\int_0^{\pi/2}\int_1^2 r\cos\varphi\cdot r^2\sin\varphi\, dr\, d\varphi\, d\theta$$

$$= 2\pi\int_0^{\pi/2}\underbrace{\sin\varphi\cos\varphi}_{\frac{1}{2}\sin(2\varphi)}\, d\varphi\cdot\int_1^2 r^3\, dr = \frac{15\pi}{4},\quad V = \frac{1}{2}\cdot\frac{4\pi}{3}\left(2^3 - 1^3\right) = \frac{14\pi}{3}$$

$\Rightarrow \bar{z} = \underline{\underline{\frac{45}{56}}} < 1 \Rightarrow$ KESKIÖ ON ULKOPUOLELLA, KOSKA $\bar{x} = \bar{y} = 0$

HUOM: KAPPALE PYSYY TASAPAINOSSA, JOS SE TUETAAN SISÄPINNAN KESKIPISTEESTÄ.
(KOSKA PAINOPISTE ALEMPANA KUIN TUKIPISTE)

EKSTRA:

> $assume\left(t > 0 \text{ and } t < \frac{\mathrm{Pi}}{2}\right)$

> $about(t)$

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Originally t, renamed t~:
  is assumed to be: RealRange(Open(0),Open(1/2*Pi))
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> $int(\mathrm{sqrt}(s^2 \cdot \cos(t)^2 + \sin(t)^2 + \sin(t)^4), s = 0..1)$

$$\frac{1}{2\cos(t\sim)}\left(\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\cos(t\sim)^4 - 3\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\cos(t\sim)^2 + \sqrt{\cos(t\sim)^4 - 2\cos(t\sim)^2 + 2}\,\cos(t\sim) + 2\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\right) \tag{1}$$

> $int\left(\%, t = 0..\frac{\mathrm{Pi}}{2}\right)$

$$\int_0^{\frac{\pi}{2}} \frac{1}{2\cos(t\sim)}\left(\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\cos(t\sim)^4 - 3\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\cos(t\sim)^2 + \sqrt{\cos(t\sim)^4 - 2\cos(t\sim)^2 + 2}\,\cos(t\sim) + 2\operatorname{arcsinh}\left(\frac{\cos(t\sim)}{\sqrt{\sin(t\sim)^2+1}\,\sin(t\sim)}\right)\right) \mathrm{d}t\sim \tag{2}$$

> $evalf(\%)$

$$1.506803134 \tag{3}$$

> $A := 4 \cdot \%$

$$A := 6.027212536 \tag{4}$$

6.
$$\begin{cases} x = \cos u \\ y = v \sin u \\ z = v \end{cases} \Rightarrow \frac{\partial \bar r}{\partial u} = -\sin u\,\bar i + v\cos u\,\bar j$$

$$\frac{\partial \bar r}{\partial v} = \sin u\,\bar j + \bar k$$

$$\bar N = \bar r_u \times \bar r_v = \begin{vmatrix} \bar i & \bar j & \bar k \\ -\sin u & v\cos u & 0 \\ 0 & \sin u & 1 \end{vmatrix} = v\cos u\,\bar i + \sin u\,\bar j - \sin^2 u\,\bar k$$

$$u = \frac{\pi}{4},\ v = \frac{1}{2} \Rightarrow \underline{\underline{\bar N = \frac{1}{2\sqrt2}\bar i + \frac{1}{\sqrt2}\bar j - \frac{1}{2}\bar k}},\quad \|\bar N\| = \sqrt{\frac18 + \frac12 + \frac14}$$

$$= \sqrt{\frac78} = \underline{\underline{\frac12\sqrt{7/2}}}$$