CIV-E1020 - Mechanics of Beam and Frame Structures –

Duration: 3h + additional 2x15 min You should return scanned hand written answers as in a contact exam in a good pdf-fquality

It is compulsory to solve only THREE (3) EXERCISES that you choose freely: only three best exercises (answers) will be graded even if the student solves four.

1) Results given without shown the logical steps needed to achieve them will be ignored even if correct.
2) Sequentially number (numeroi juoksevasti) your answer papers 1(*n*) ... *n*(*n*), where *n* =total number of pages
3) Write readably your name, family name and student number.
4) Name the pdf-file: `Studdent_ID_Name_Date.pdf` Make a good quality scanned pdf
5) All additional material, like listings, graphs can be appended as pdfs to the answer
6) Please check the physical units of your answers.

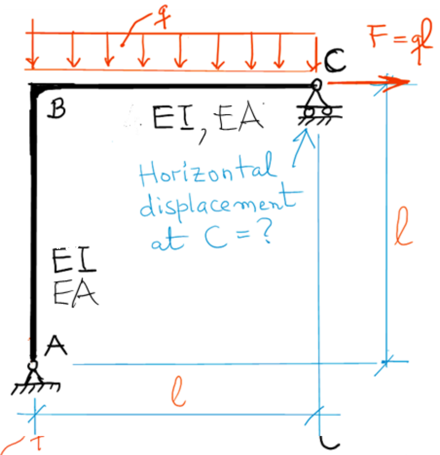
Exam, points		Grade
≥13,75		5
12,75	13,5	4
9,75	12,5	3
8,25	9,5	2
7	8	1
<7		fail=0

Examination 26/10/2021

The material is linear elastic in all the structures below

1. Use the dummy unit-load theorem (or method) and determine the horizontal displacement at roller **C** [3 point]. Account for both the effects of bending and axial forces when computing displacement. Ignore the shear effects. [overall = 5 points]

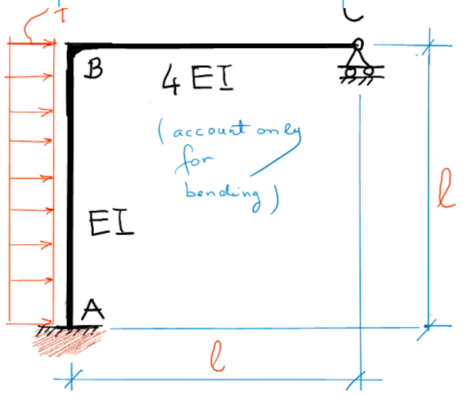
$$EI = \alpha EA \cdot l^2$$



Hints: is it statically determined? Determine support reactions
Then determine and draw accurately the bending moment [1 point] and axial force [1 point] diagrams

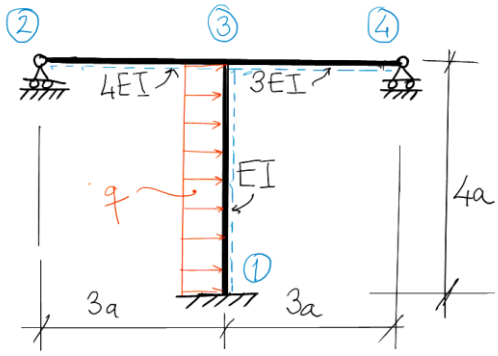
2. Use the general force method and
a) determine the bending moment at support *A* [3 points] and draw accurately the bending moment diagram [1 point]. *Account only for effects of bending*

b) Determine the support reaction at *C* (value and direction). [1 p].

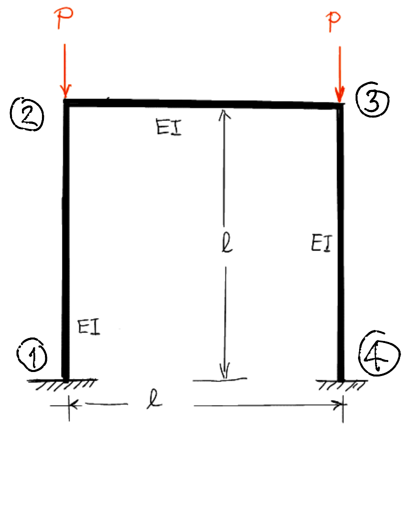


3. Use Slope-Deflection Method and
a) **determine the bending moment** at clamping support *1* [4 points]
b) use results from question *a*) and determine the horizontal displacement at roller 4 [1 point] **(all other methods are welcomed for evaluating the displacement)**

Hint a): If you wish you can use the stiffness-moment relation for hinged beams where appropriate



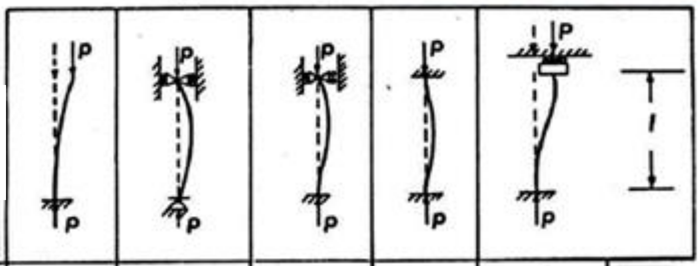
4. Buckling of sway-frames
The Frame is loaded symmetrically with by two concentrated loads *P*.
Use Slope-Deflection Method and 1) derive the explicit expression, in terms of Berry's stability functions, of the needed **criticality condition for determining the critical buckling load** *P* [3 points]. **Hint**: assume anti-symmetric buckling mode
2) solve numerically for the value of the buckling load *P* [1 point].
3) Give a **bracket** for the value of **buckling load** using cleverly the Euler's basic cases (see tables in the formulary) [1 point].



Euler's basic buckling cases

Eulerin perusnurjahdus

$$P_{cr} = \mu \frac{\pi^2 EI}{\ell^2}$$



Tapaus	1	2	3	4	5
μ	0,25	1	2,046	4	1

$$M_{ij} = A_{ij}\phi_{ij} + B_{ij}\phi_{ji} - C_{ij}\psi_{ij} + \bar{M}_{ij} \qquad M_{ij}^0 = A_{ij}^0\varphi_{ij} - C_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

Beam-column with constant flexural rigidity:

$$A_{ij} = A_{ji} = \frac{2\psi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L} \qquad B_{ij} = B_{ji} = \frac{\phi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}$$

$$C_{ij} = A_{ij} + B_{ij} \qquad \underbrace{A_{ij}^0 = C_{ij}^0 = \frac{1}{\psi(kL)}\frac{3EI}{L},}_{\text{one end is hinged } B^0_{ij} = B^0_{ji} = 0} \qquad kL \equiv L\sqrt{\frac{P}{EI}}$$

Berry's functions:

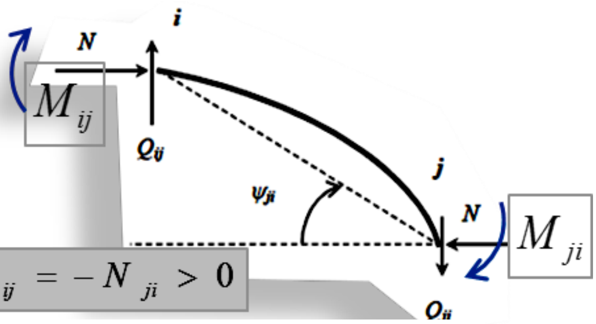
Olkoon $\lambda \equiv kL$, $\qquad \lambda \equiv kL$

Puristettu sauva:

Compression: $\phi(\lambda) = \frac{6}{\lambda}\left(\frac{1}{\sin\lambda} - \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(\frac{1}{\lambda} - \frac{1}{\tan\lambda}\right)$ ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(\tan\frac{\lambda}{2} - \frac{\lambda}{2}\right)$,

Vedetty sauva:

Extension: $\phi(\lambda) = \frac{6}{\lambda}\left(-\frac{1}{\sinh\lambda} + \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(-\frac{1}{\lambda} + \frac{1}{\tanh\lambda}\right)$ ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(-\tanh\frac{\lambda}{2} + \frac{\lambda}{2}\right)$,



$$N \equiv -N_{ij} = -N_{ji} > 0$$

$$Q_{ij} = Q_{ij}^0 - (M_{ij} + M_{ji})/L - N\psi_{ij}$$

$$\overline{M}_{12} \equiv MK_1$$

$$\overline{M}_{ij}, \overline{M}_{ji}$$

N:o	Kuormitus	Kiinnitysmomentit: MK_1, MK_2
1	q, L	$MK_1 = -\frac{qL^2}{12},\ MK_2 = \frac{qL^2}{12}$
2		

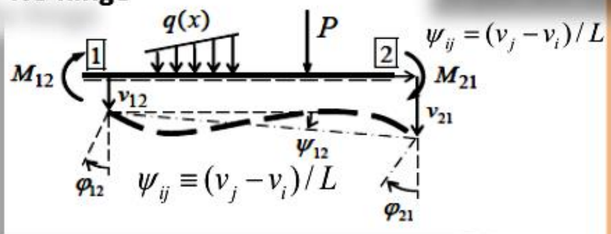
$$\overline{M}_{ij} \qquad \overline{M}_{ji}$$

N:o	Kuormitus	Nivel oikeassa päässä: MK_1^0	Nivel vasemmassa päässä: MK_2^0
1	q, L	$MK_1^0 = -\frac{qL^2}{8}$	$MK_2^0 = \frac{qL^2}{8}$

The stiffness equation relating the end-moments to the end-displacements

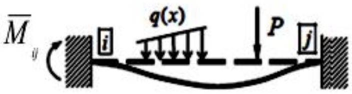
If you are using lecture's notations

No hinge



$$M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + \overline{M}_{ij}, \quad i \neq j$$

$$a_{ij} = \frac{4EI}{L}, \quad b_{ij} = \frac{2EI}{L}, \quad c_{ij} = \frac{6EI}{L}$$ (*EI*-constant)



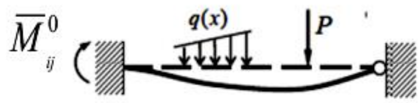
Fixed end-moment resulting from external mechanical loading, look from tables

One node is hinged

The is a superscript "0" means that the support at end *j* is hinged

$$M_{ij}^0 = a_{ij}^0\varphi_{ij} - c_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

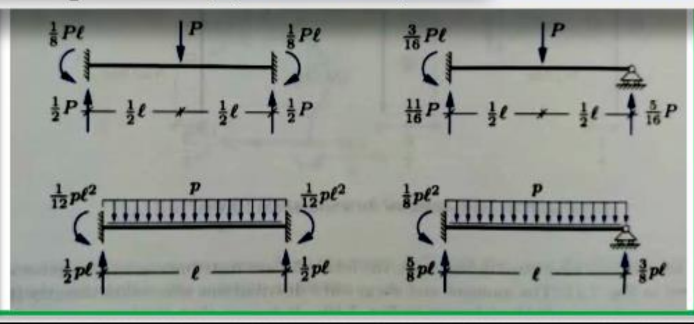
$$a_{12}^0 = c_{12}^0 = \frac{3EI}{L} \qquad \psi_{ij} = (v_j - v_i)/L$$



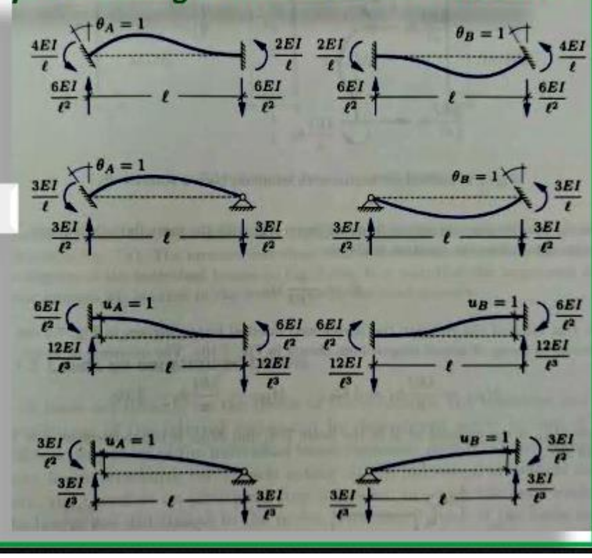
Fixed end-moment resulting from external mechanical loading, look from tables

If you are using Krenk's textbook notations

Bending Moments (pay attention to the sign convention to convert to Fixed-End-Moments)



If you are using Krenk's textbook notations



Maxwell-Mohr integrals table

TABLEAU DES INTEGRALES $\int_0^{\ell} {}^{i}M\,{}^{k}M\,dx$

${}^{k}M$ \ ${}^{i}M$	c, ℓ	c, ℓ	d, ℓ	c, d, ℓ	2ème deg. c, ℓ
a, ℓ	$ac\ell$	$\frac{1}{2}ac\ell$	$\frac{1}{2}ad\ell$	$\frac{1}{2}a\ell(c+d)$	$\frac{2}{3}ac\ell$
a, ℓ	$\frac{1}{2}ac\ell$	$\frac{1}{3}ac\ell$	$\frac{1}{6}ad\ell$	$\frac{1}{6}a\ell(2c+d)$	$\frac{1}{3}ac\ell$
b, ℓ	$\frac{1}{2}bc\ell$	$\frac{1}{6}bc\ell$	$\frac{1}{3}bd\ell$	$\frac{1}{6}b\ell(c+2d)$	$\frac{1}{3}bc\ell$
a, b, ℓ	$\frac{1}{2}(a+b)c\ell$	$\frac{1}{6}(2a+b)c\ell$	$\frac{1}{6}(a+2b)d\ell$	$\frac{\ell}{6}[a(2c+d) + b(c+2d)]$	$\frac{1}{3}(a+b)c\ell$
2ème deg. a, ℓ	$\frac{1}{3}ac\ell$	$\frac{1}{4}ac\ell$	$\frac{1}{12}ad\ell$	$\frac{1}{12}a\ell(3c+d)$	$\frac{1}{5}ac\ell$
2ème deg. a, ℓ	$\frac{2}{3}ac\ell$	$\frac{5}{12}ac\ell$	$\frac{1}{4}ad\ell$	$\frac{1}{12}a\ell(5c+3d)$	$\frac{7}{15}ac\ell$