Examination 27.10.2021 Lassila

Please answer to all five (5) questions. You may answer using English, Finnish or Swedish.

- 1. New jobs arrive at a lossy queueing system consisting of 2 parallel servers and 4 waiting places with a rate of 4 jobs/ms. Some of these are blocked because all waiting places are full. On the other hand, it has been measured that jobs are exiting the system after service at a rate of 3 jobs/ms. In addition, the mean total delay of a job through the system is 1 ms and the mean number of waiting jobs is 2.
 - a) What is the probability that an arriving job is blocked? (2p)
 - a) What is the average waiting time of jobs? (2p)
 - b) What is the average number of jobs in service? (2p)
- 2. Consider a queueing system with two parallel servers. The service times are independent and obey the exponential distribution with parameters $\mu_1 = 1$ (server 1) and $\mu_2 = 2$ (server 2). The system is empty at time 0. The first customer arrives to the system at time 1 in server 1 and the second customer arrives at time 3 in server 2, respectively. No other customers enter the system. In addition, it is known that both customers are still in the system at time 4. Let Z_1 denote the time at which the customer with the shorter service time leaves the system. Correspondingly, let Z_2 denote the time at which the customer with the longer service time departs. Thus, $Z_2 > Z_1 > 4$. Determine the mean values $E[Z_1]$ and $E[Z_2]$. (6p)
- 3. Consider data traffic at the packet level. Packets arrive to the buffer of the transmission link from incoming links according to independent Poisson processes with the following rates: 20, 35, 10 and 15 packets/millisecond. The average packet length is exponentially distributed with mean 10 000 bits. Packets from the buffer are served by a link with capacity 1 Gbit/s. Model the system as a pure single server queuing system with an infinite buffer and answer the following.
 - (a) Draw the state transition diagram for the Markov process X(t) representing the number of packets in the system. What is the Kendall's notation for the model? (1p)
 - (b) Derive the equilibrium distribution for the system. Is there any condition for existence of the equilibrium distribution, i.e., on the stability? (3p)
 - (c) Derive the expression for the mean delay of packets? What is the mean delay with these parameters? (2p)
- 4. Consider elastic data traffic carried by a 100-Mbps link in a packet switched network. New flows arrive according to a Poisson process at rate 100 flows per second, and the sizes of files to be transferred are independently and exponentially distributed with mean 1 Mbit. Flows share the capacity of the 100 Mbps link perfectly fairly according to the PS discipline. However, each flow has additionally an access link with maximum rate 50 Mbps, which limits the maximum service rate of an individual flow. In the system, new flow arrivals are rejected if there are already 3 flows in the system. You can model the system as an M/M/2/3-PS queue. Let X(t) denote the number of ongoing flows at time t, which is a Markov process.
 - (a) Derive the equilibrium distribution of X(t). (3p)
 - (b) What is the throughput of a flow (that is not rejected)? (3p)

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- 5. Consider a system of two components in parallel. The failure rate of component 1 is $\lambda_1 = 3$ and the repair rate is $\mu_1 = 1$ and correspondingly for component 2 the failure rate is $\lambda_2 = 2$ and the repair rate is $\mu_2 = 2$. Assume further that the failure and repair times are independent and obey the exponential distribution (with the respective parameters).
 - a) Draw the state transition diagram for the Markov process representing the state of the whole system. (2p)
 - b) Derive the equilibrium distribution of the system. What is the fraction of time the system is functional (mean availability)? Hint: Write down and solve balance equations directly using the given numerical values of parameters, instead of solving symbolically. (3p)
 - c) What is the mean down time (MDT) of the system? (1p)