

Please follow the instruction given on the exam page. Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified.

**PROBLEM 1** Find the derivative of  $f(x) = \cos x$  using Newton's Quotient with the help of the identity

$$\cos(\xi + k) = \cos(k)\cos(\xi) - \sin(k)\sin(\xi).$$

PROBLEM 2 (4p) Find the Maclaurin series of (Taylor series about the origin) of degree 3

$$f(x) = \sin(x + \pi/4).$$

(2p) Estimate the error at  $x = \pi/4$ . Report the true error  $T_3(f)(\pi/4) - f(\pi/4)$  and the theoretical estimate and comment on the accuracy of the latter.

**PROBLEM 3** Compute the integral

$$\int_0^4 \sin\left(\pi\sqrt{x}\,\right) \, dx,$$

using substitution  $x = t^2$ .

PROBLEM 4 Compute the integral

$$\int_0^{2\sqrt{\pi}} x \ln x \, dx.$$

PROBLEM 5 Solve the 2nd order ordinary differential equation

$$\begin{cases} y'' + 9y = \cos 2t \\ y(0) = 1, \\ y'(0) = 0. \end{cases}$$