

Use of calculators is not allowed in the exam.

You are allowed to bring with you a single one-sided A4 “cheat sheet”, **personally handwritten by you**. (NO photocopies, NO printouts, NO computer type-set text.) Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.

Note: If you have not completed your computerised home assignments, your exam will not be graded.

1. (a) Design a deterministic finite automaton that recognises the language

$$L = \{a^n \mid n \text{ is divisible by 2 or 3 (or both)}\}.$$

(The value 0 is considered to be divisible by any number.)

8 points

- (b) Give a regular expression that describes the language L in part (a).

6 points

2. Is the language

$$L = \{0^m 1^n \mid m \leq n\}$$

- (a) regular, (b) context-free but nonregular, (c) decidable but not context-free, (d) undecidable?
Give a *precise justification* for your answer. 15 points

3. A language class \mathcal{C} is said to be *closed under complement*, if for every $L \in \mathcal{C}$ also $\bar{L} \in \mathcal{C}$.

- (a) Show that the class of regular languages is closed under complement. 7 points

- (b) Show that the class of context-free languages is not closed under complement. (*Hint:* You may use the information from the course that the language $L = \{a^n b^n c^n \mid n \geq 0\}$ is not context-free.) 8 points

4. Which of the following claims are true and which are false? (No proofs are needed, just indicate your choice by the letter T or F.)

- (a) The union of any two regular languages is context-free.
(b) Every language that can be recognised by a nondeterministic pushdown automaton can be generated by a context-free grammar.
(c) Every language that can be recognised by a deterministic pushdown automaton can be described by a regular expression.
(d) There exist undecidable context-free languages.
(e) Nondeterministic Turing machines recognise (“semidecide”) exactly the recursively enumerable languages.
(f) The language $\{a^n b^n \mid n \geq 0\}$ can be recognised by a nondeterministic finite automaton.
(g) The complement of every decidable language is semidecidable.
(h) The computation of a deterministic Turing machine terminates on every input.

16 points

Total 60 points