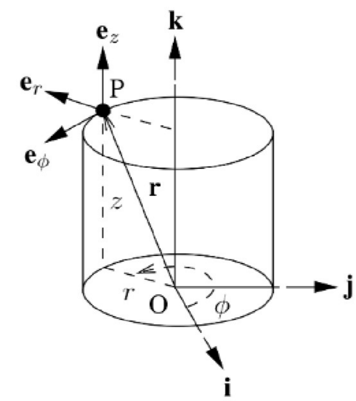
Note: this exam has two pages and there are five problems (max 30 points).

Problem 1. Give a detailed derivation of the following formulas for the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the point P (see figure) in cylindrical coordinate system (coordinates r, ϕ, z):

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\phi}\mathbf{e}_\phi + \dot{z}\mathbf{e}_z$$
$$\mathbf{a} = (\ddot{r} - r\dot{\phi}^2)\mathbf{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\mathbf{e}_\phi + \ddot{z}\mathbf{e}_z.$$

Start from the position vector $\mathbf{r} = r\mathbf{e}_r + z\mathbf{e}_z$ of the point P.

(6 p.)

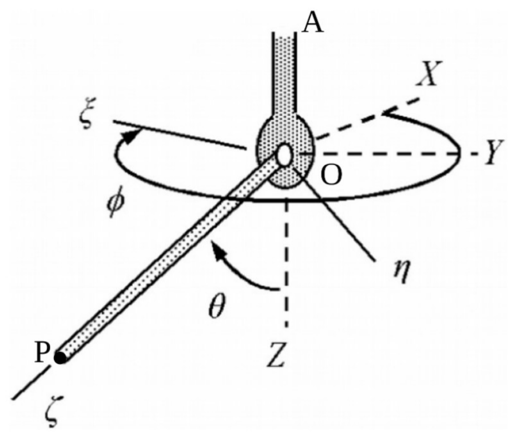


Problem 2. The rotating pendulum of the figure is made of rigid rod OP (length l), which is hinged at point O to a peg OA. The peg and the rod are rotating with constant rate $\dot{\phi}$ around Z axis. Simultaneously, the rod OP is tilting with constant rate $\dot{\theta}$ around ξ axis.

(a) Use the equations of relative velocity and find the velocity of the point P of rod. **(3 p.)**

(b) Find the acceleration of point P. **(3 p.)**

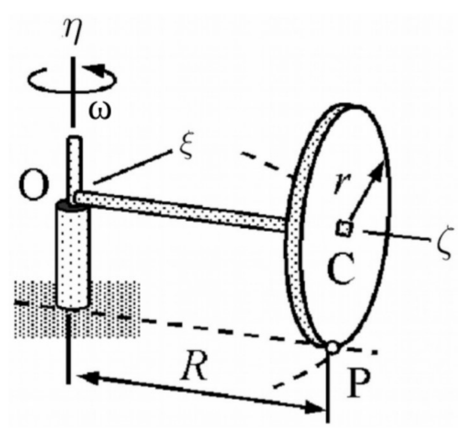
Present your solutions in $\xi\eta\zeta$-frame which is fixed to the rod and has its origin at O. XYZ-frame is fixed.



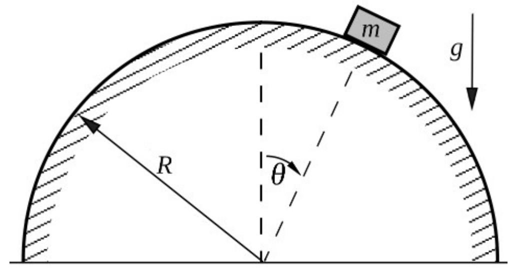
Problem 3. A rigid body consisting of a circular disk (radius r) and axle OC (length R) is hinged at point O and rolling on a horizontal floor without slipping. The angular velocity of the rigid body around η axis is ω, and it is constant. The elements of the diagonal inertia matrix of the body are $J_{\zeta\zeta} = I$ and $J_{\xi\xi} = J_{\eta\eta} = I_{\mathrm{O}}$.

(a) Find the angular momentum $\mathbf{l}$ of the rigid body. Present your solution in $\xi\eta\zeta$-frame. **(4 p.)**

(b) Calculate the torque produced by the rigid body at the origin O of $\xi\eta\zeta$-frame.**(2 p.)**



Problem 4. A particle of mass m slides downwards on a frictionless, fixed half-sphere (radius R) as shown in the figure. Derive expressions for the generalized force Q_θ and the kinetic energy T using the generalized coordinate θ, and find the equations of motion for the system. Use the Lagrange's method, the form based on the kinetic energy T and generalized forces Q_j. Assume that the particle remains in contact with the sphere throughout the full motion. **(6 p.)**



Problem 5. The figure shows a pulley system consists of two masses (masses m_1 and m_2). The rope is unstretchable (rope length l = constant), massless and friction prevents slipping between the wheel and the rope. Find the Lagrangian L of the system and derive the equations of motion using the generalized coordinate x. The mass of the pulley-wheel is m and moment of inertia about center of mass is $I = \frac{1}{2}mR^2$. Assume that the pulley rotates frictionlessly. **(6 p.)**