Matrix Algebra
MS-A0001
Hakula/Nyman
Course Exam, 14.12.2021

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Please follow the instruction given on the exam page. Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified. There are five problems in the exam.

PROBLEM 1

a) Show that if P is a permutation matrix, then $\det(P) = 1$ or -1.

b) Let $\mathbf{w} = \mathbf{u} \times \mathbf{v}$. Show that $\|\mathbf{w}\|^2 = \|\mathbf{u}\|^2\|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2$.

c) Let $u, v \in \mathbb{R}^3$. Show that $\det(uv^{\mathrm{T}}) = 0$.

PROBLEM 2 Find the decomposition $A = LDL^{\mathrm{T}}$, where

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & \alpha \end{pmatrix},$$

for all $\alpha \in \mathbb{R}$.

PROBLEM 3 Solve the linear system of equations $Ax = b$ using Gaussian elimination, when

$$A = \begin{pmatrix} 1 & 3 & 5 & -2 \\ 3 & -2 & -7 & 5 \\ 2 & 1 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 11 \\ 0 \\ 3 \end{pmatrix}.$$

PROBLEM 4 Find the angle between the eigenvectors of

$$\begin{pmatrix} 1 & 1 \\ 0 & 1+t \end{pmatrix}$$

as a function of the parameter t. What is the relation between this angle and the linear independence of the eigenvectors?

PROBLEM 5 Let A be a 3×3-matrix. Its eigenvalues are

$$\lambda_1 = 1, \quad \lambda_2 = 1/2, \quad \lambda_3 = 1/3,$$

and the corresponding eigenvectors

$$v_1 = (1, 1, 1)^{\mathrm{T}}, v_2 = (0, 1, 1)^{\mathrm{T}}, v_3 = (0, 0, 1)^{\mathrm{T}}.$$

Find $\lim_{k\to\infty} A^k$.