CIV-E1020 - Mechanics of Beam and Frame Structures –

# Retake exam. 16/12/2021

Duration: 3h + additional 2x15 min You should return scanned hand written answers as in a contact exam in a good pdf-quality

| Exam. points | | Grade |
|---|---|---|
| ≥13,75 | | 5 |
| 12,75 | 13,5 | 4 |
| 9,75 | 12,5 | 3 |
| 8,25 | 9,5 | 2 |
| 7 | 8 | 1 |
| < 7 | | fail=0 |

**It is compulsory to solve only THREE (3) EXERCISES that you choose freely: only three best exercises (answers) will be graded even if the student solves four.**

1) Results given without shown the logical steps needed to achieve them will be ignored even if correct.
2) Sequentially number (numeroi juoksevasti) your answer papers 1(*n*) ... *n*(*n*), where *n* =total number of pages
3) Write readably your name, family name and student number.
4) Name the pdf-file: `Studdent_ID_Name_Date.pdf` Make a good quality scanned pdf
5) All additional material, like listings, graphs can be appended as pdfs to the answer
6) Please check the physical units of your answers.

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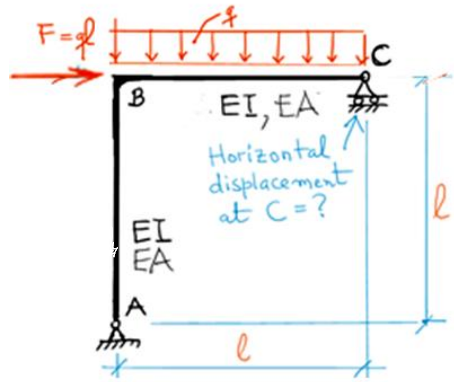
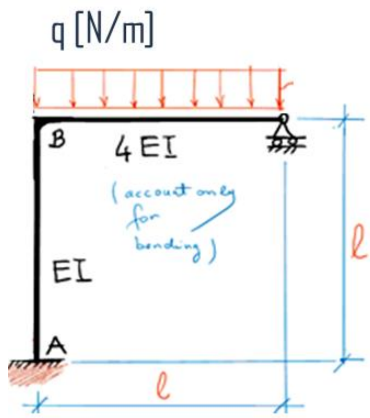
The material is linear elastic in all the structures below



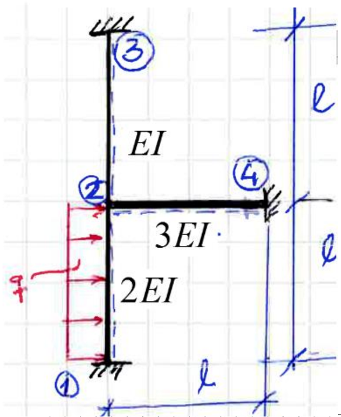

1. **Use the dummy unit-load theorem** (or method) and determine the horizontal displacement at roller **C** [3 point]. Account for both the effects of bending and axial forces when computing displacement. Ignore the shear effects. [overall = 5 points] $EI = \alpha EA \cdot \ell^2$

Hints: is it statically determined? Determine support reactions
Then determine and draw accurately the bending moment [1 point] and axial force [1 point] diagrams

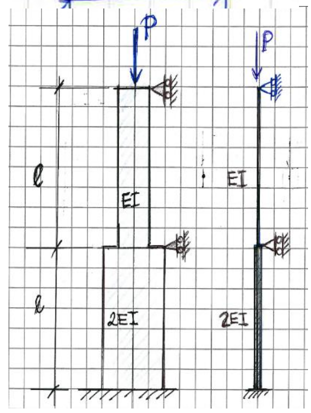



2. Use the general force method and
*a*) determine the bending moment at support *A* [3 points] and draw accurately the bending moment diagram [1 point]. *Account only for effects of bending*

b) Determine the support reaction at *the roller* (value and direction). [1 p].




3. Use Slope-Deflection Method and
*D*etermine the bending moment at clamping support *1* [4 points]
Draw accurately the bending moment diagram [1 point]

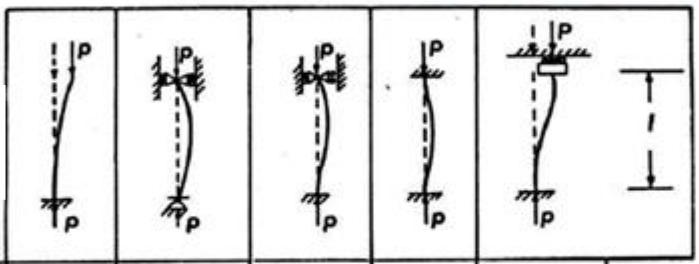



**4.** Elastic **Buckling**
A linear elastic continuous column is loaded centrically by a load *P* (see Fig.)
Use Slope-Deflection Method and **1**) derive the explicit expression, in terms of Berry's stability functions, of the needed **criticality condition for determining the critical buckling load** [3 points].
**2) solve numerically for the value of the buckling load** *P* [1 point].
**3)** (cross-check=) Give a **bracket** for the value of **buckling load** using cleverly the Euler's basic cases (see tables in the formulary) [1 point].

## Euler's basic buckling cases

Eulerin perusnurjahdus

$$P_{cr} = \mu \frac{\pi^2 EI}{\ell^2}$$




| Tapaus | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| μ | 0.25 | 1 | 2.046 | 4 | 1 |

$$M_{ij} = A_{ij}\phi_{ij} + B_{ij}\phi_{ji} - C_{ij}\psi_{ij} + \bar{M}_{ij} \qquad M_{ij}^0 = A_{ij}^0\varphi_{ij} - C_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

**Beam-column with constant flexural rigidity:**

$$A_{ij} = A_{ji} = \frac{2\psi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L} \qquad B_{ij} = B_{ji} = \frac{\phi(kL)}{4\psi^2(kL) - \phi^2(kL)}\frac{6EI}{L}$$

$$C_{ij} = A_{ij} + B_{ij} \qquad \underbrace{A_{ij}^0 = C_{ij}^0 = \frac{1}{\psi(kL)}\frac{3EI}{L},}_{\text{one end is hinged } B_{ij}^0 = B_{ji}^0 = 0} \qquad kL \equiv L\sqrt{\frac{P}{EI}}$$

**Berry's functions:**

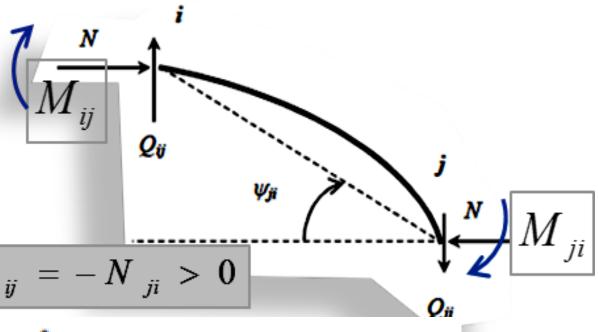
Olkoon $\lambda \equiv kL$, $\qquad \lambda \equiv kL$

*Puristettu sauva:*

**Compression:** $\phi(\lambda) = \frac{6}{\lambda}\left(\frac{1}{\sin\lambda} - \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(\frac{1}{\lambda} - \frac{1}{\tan\lambda}\right)$, ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(\tan\frac{\lambda}{2} - \frac{\lambda}{2}\right)$,

*Vedetty sauva:*

**Extension:** $\phi(\lambda) = \frac{6}{\lambda}\left(-\frac{1}{\sinh\lambda} + \frac{1}{\lambda}\right)$, $\psi(\lambda) = \frac{3}{\lambda}\left(-\frac{1}{\lambda} + \frac{1}{\tanh\lambda}\right)$, ja $\chi(\lambda) = \frac{24}{\lambda^3}\left(-\tanh\frac{\lambda}{2} + \frac{\lambda}{2}\right)$,




$$N \equiv -N_{ij} = -N_{ji} > 0$$

$$Q_{ij} = Q_{ij}^0 - (M_{ij} + M_{ji})/L - N\psi_{ij}$$

$$\overline{M}_{12} \equiv MK_1$$

$$\overline{M}_{ij}, \overline{M}_{ji}$$

| N:o | Kuormitus | Kiinnitysmomentit: $MK_1$ $MK_2$ |
|---|---|---|
| 1 | $q$, $L$ | $MK_1 = -\frac{qL^2}{12}$, $MK_2 = \frac{qL^2}{12}$ |
| 2 | | |

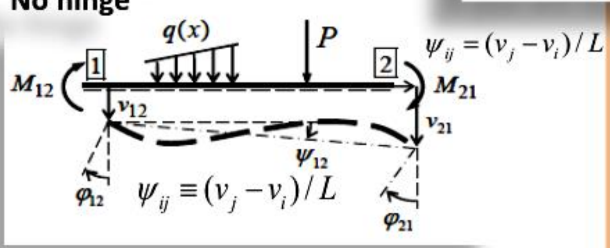
$$\overline{M}_{ij} \qquad \overline{M}_{ji}$$

| N:o | Kuormitus | Nivel oikeassa päässä: $MK_1^0$ | Nivel vasemmassa päässä: $MK_2^0$ |
|---|---|---|---|
| 1 | $q$, $L$ | $MK_1^0 = -\frac{qL^2}{8}$ | $MK_2^0 = \frac{qL^2}{8}$ |

**The stiffness equation relating the end-moments to the end-displacements**

**If you are using lecture's notations**

**No hinge**




$$M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + \overline{M}_{ij}, \quad i \neq j$$

$$a_{ij} = \frac{4EI}{L}, \quad b_{ij} = \frac{2EI}{L}, \quad c_{ij} = \frac{6EI}{L}$$ (*EI*-constant)

$\overline{M}_{ij}$


Fixed end-moment resulting from external mechanical loading, look from tables

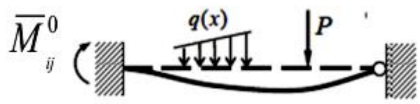
**One node is hinged**

The is a superscript "0" means that the support at end *j* is hinged

$$M_{ij}^0 = a_{ij}^0\varphi_{ij} - c_{ij}^0\psi_{ij} + \overline{M}_{ij}^0$$

$$a_{12}^0 = c_{12}^0 = \frac{3EI}{L} \qquad \psi_{ij} = (v_j - v_i)/L$$

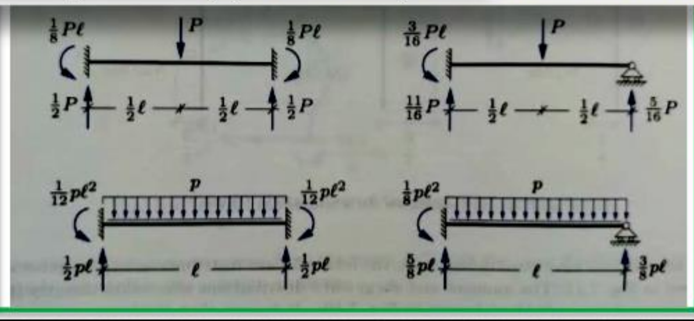
$\overline{M}_{ij}^0$



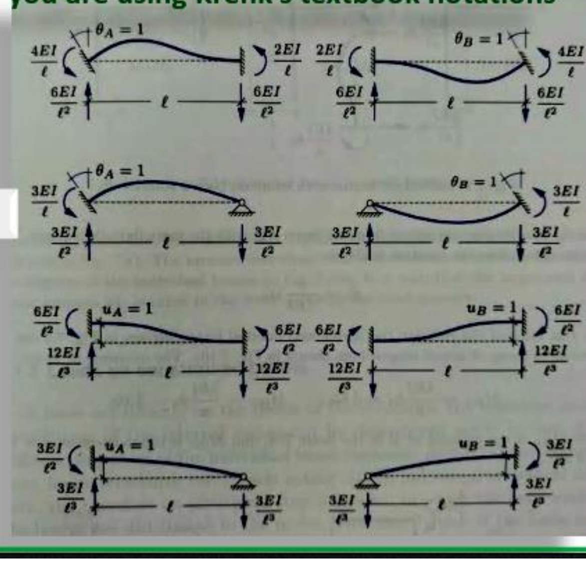

Fixed end-moment resulting from external mechanical loading, look from tables

**If you are using Krenk's textbook notations**

**Bending Moments** (pay attention to the sign convention to convert to Fixed-End-Moments)



**If you are using Krenk's textbook notations**



# Maxwell-Mohr integrals table

TABLEAU DES INTEGRALES $\int_0^\ell {}^iM\,{}^kM\,dx$

| ${}^kM$ \ ${}^iM$ | c, ℓ | c, ℓ | d, ℓ | c, d, ℓ | 2ème deg. c, ℓ |
|---|---|---|---|---|---|
| a, ℓ | $ac\ell$ | $\frac{1}{2}ac\ell$ | $\frac{1}{2}ad\ell$ | $\frac{1}{2}a\ell(c+d)$ | $\frac{2}{3}ac\ell$ |
| a, ℓ | $\frac{1}{2}ac\ell$ | $\frac{1}{3}ac\ell$ | $\frac{1}{6}ad\ell$ | $\frac{1}{6}a\ell(2c+d)$ | $\frac{1}{3}ac\ell$ |
| b, ℓ | $\frac{1}{2}bc\ell$ | $\frac{1}{6}bc\ell$ | $\frac{1}{3}bd\ell$ | $\frac{1}{6}b\ell(c+2d)$ | $\frac{1}{3}bc\ell$ |
| a, b, ℓ | $\frac{1}{2}(a+b)c\ell$ | $\frac{1}{6}(2a+b)c\ell$ | $\frac{1}{6}(a+2b)d\ell$ | $\frac{\ell}{6}[a(2c+d) + b(c+2d)]$ | $\frac{1}{3}(a+b)c\ell$ |
| 2ème deg. a, ℓ | $\frac{1}{3}ac\ell$ | $\frac{1}{4}ac\ell$ | $\frac{1}{12}ad\ell$ | $\frac{1}{12}a\ell(3c+d)$ | $\frac{1}{5}ac\ell$ |
| 2ème deg. a, ℓ | $\frac{2}{3}ac\ell$ | $\frac{5}{12}ac\ell$ | $\frac{1}{4}ad\ell$ | $\frac{1}{12}a\ell(5c+3d)$ | $\frac{7}{15}ac\ell$ |