Course Name: Production Planning and Optimization

Course Code: CHEM-E7151

Retake Exam, 25.10.2021, 09.00-13.00

Exam Method: MyCourses Assignment activity with a time restriction.

Write your full name and student number on the top of your answer sheets.

Submit your answers to the Submission folder named "Course Exam 2021".

All questions need to be answered (total of 20 points).

Question 1. *(2 points)*

Chemco company produces three types of chemicals: X, Y, and Z. These products are manufactured by two operations 1 and 2. Operation 1 costs 5 EUR/hour and produces 3 units of X, 2 units of Y, and 3 units of Z in one hour. Operation of process 2 costs 2 EUR/hour and yields 2 units of X and 1 unit of Z in one hour. According to demand requirements, at least 18 units of X, 4 units of Y, and 12 units of Z should be produced per day. The company wants to provide a daily manufacturing plan that minimizes the total cost of fulfilling the daily demands.

a) Briefly answer:

- Using what type of model can this problem be formulated**?** (0.25 points)

- Which approach/s can be used to find a solution for this problem**?** (0.25 points)

b) Formulate the given problem. **Do not solve it!** (1.5 points)

Question 2. *(4 points)*

A company produces two products A and B using three resources: labor, raw material R, and raw material M. The company has only 12 person-hours of labor, 18 kg of material R, and 24 kg of material M available each day. One unit of product A requires 5 person-hours of labor, 2 kg of R, and 5 kg of M. A unit of product B needs 1 person-hour of labor, 3 kg of R, and 3 kg of M. The company can sell as much of each product as it can produce and gain a profit of 6 EUR per unit of A and 2 EUR per unit of B.

a) Formulate the problem analytically. (1 point)

b) Use the **Simplex** method to maximize the company's sales revenues and find the optimal amount of products to be produced daily. (3 points)

Question 3. *(3 points)*

An oil-refining company wants to invest 4 million euros in building refinery sites. Three sites are under consideration. The amount of earned revenue depends on the amount of money invested in each refinery, as shown in Table 1. Use **Dynamic Programming** to determine an investment plan that will maximize the company's revenue gained from these three sites. Assume that the amount invested in each refinery must be an exact multiple of 1 million EUR (MEUR).

Table 1.

Investment amount (MEUR)	**Revenue (MEUR)**		
	Refinery 1	**Refinery 2**	**Refinery 3**
0	*8*	*6*	*5*
1	*11*	*9*	*9*
2	*12*	*13*	*10*
3	*13*	*15*	*15*
4	*14*	*17*	*17*

Question 4. *(5 points)*

A specialty steel plant produces and delivers two products to a car factory, steel plates (X) for being used e.g. for the roof and doors and beams (Y) used for supporting constructions. For continuity, it is important for the plant to be able to produce amounts that can be delivered daily to the car manufacturing line, which is located close to the steel plant. For simplicity, let's assume that a unit of X and Y is equivalent to the amount needed for one car. The car factory could also order beams from other steel plants, but the considered plant is unique in being able to produce the needed plate steel. Therefore, the steel plant practically always makes more units of product X than product Y. To reduce its dependency on other steel plants, the car factory has agreed that the steel plant produces at least 30% beams of the amount of plates (i.e. for every 100 units of plates, it must produce at least 30 units of beams).

Producing a steel plate (X) requires 150 kg, of raw steel, 2 MWh of electricity and takes 10 minutes labor time. A beam (Y) again requires 210 kg raw steel, 1 MWh electricity and 8 minutes labor. The steel plant operates between 6-20 (two shifts) and has always 10 workers at the factory. The goal is to maximize the profit when we know that the plates cost 1200 EUR and the beams 720 EUR. What is the best daily production strategy when we can consume 1500 GWh during a day and have 150 tons of raw steel available per day?

a) **Formulate** the problem analytically as an **integer programming** problem. (2 points)

b) **Solve** this model by using the **graphical** method **without integer restrictions** (relaxed LP problem). What special can you observe about this particular problem? (1 point)

c) Perform the **Branch and Bound** algorithm (draw the search tree) and **compare** the obtained optimum solution with the relaxed linear programming model solved in part b). You can solve the LP-steps either manually or using e.g. GAMS (in which case submit the code). (2 points)

Question 5 *(4 points)*

A production process has three production steps:

1. Equipment M1 takes raw material R1 in and after processing it for 1 hour, releases an intermediate I1. The capacity of M1 is 90 tons.
2. The second step has two alternative equipment:
   a. Equipment M2 consumes 40 tons of the intermediate product I1 and produces end product P1 in 1,5 hours.
   b. Equipment M3 takes 50 tons of I1 and produces end product P2 in 2 hours.
3. A packaging machine M4 can be used for both end products P1 and P2. A batch of P1 (40 tons) takes 1 hours to pack and this produces export product E1 (which can be delivered & sold), and for a batch of P2 (50 tons) the time is 1,5 hours producing E2.

In order to optimize the process:

a) Build the **RTN graph** (tasks, resources) and **define the parameters** $\mu_{i,r,\theta}$, $\nu_{i,r,\theta}$, δ, τ_i analytically (pen & paper or e.g. Excel sheet). (2 points)

b) Solve the scheduling problem in **GAMS** for one day (24 hours) maximizing the revenues when the price is 2 EUR/ton for product 1 (E1) and 3 EUR/ton for product 2 (E2). (2 points)

c) How could we avoid having too much unused material at the end? (optional)

Question 6. *(2 points)*

Let us consider a two-stage batch process with a demand to produce five tasks. Stage-specific durations and due dates of the tasks are listed in Table 1. Each stage has only one processing unit, which can process only one task at the time. No changeover times between tasks are considered. The processing of a task at Stage 2 can start only after its processing has finished at Stage 1.

a. Use the 'earliest due date first' heuristic to define the dispatching sequence for the tasks.
b. The dispatching is started at $t = 0$ h. The processing of tasks at Stages 1 and 2 is started as early as possible. Draw the Gantt chart of the schedule.
c. Determine the finishing times of the five tasks and the total tardiness of the schedule.

Table 1

Task	Due date [h]	Processing duration at Stage 1 [h]	Processing duration at Stage 2 [h]
1	35	6	4
2	25	5	11
3	15	8	6
4	20	6	3

5	40	5	6