ELEC-C9420 Introduction to Quantum Technology, Fall 21
Midterm exam 2, part B, 16.12.2021
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Problem B1

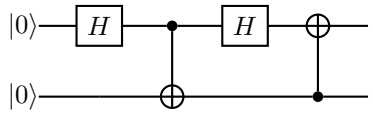
Consider a quantum particle in a 1D harmonic potential $U(x) = \frac{1}{2}kx^2$. The particle has energy eigenstates $|n\rangle$, $n = 0, 1, 2, \ldots$, with corresponding energy eigenvalues $E = \hbar\omega(n + \frac{1}{2})$, where $\omega = \sqrt{k/m}$ is the characteristic angular frequency of the oscillator. At time $t = 0$ the particle is in the state

$$|\phi(0)\rangle = \frac{1}{\sqrt{3}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{6}}|2\rangle\,.$$

a) What is the probability to detect the system in its ground state at $t = 0$? (2p)
b) Compute the expectation value of energy for the particle at $t = 0$. (2p)
c) Find the state vector $|\phi(T)\rangle$ after the system is let to evolve for time T. (2p)
d) Let's imagine the energy of the particle is measured at time T, and found to be $\frac{3}{2}\hbar\omega$. The system is then let to evolve for time T'. What is the probability to find the particle in its ground state at time $t = T + T'$ in this case? (2p)

Problem B2

Consider the following quantum circuit.



a) What is the output state of the circuit? (2p)
b) Are the qubits entangled in the output state? (2p)
c) What are the probablities to measure different qubit values in the output state? (2p)
d) Find the expectation value for the sum of the qubit values in the output state. (2p)