Aalto University

MS-A0211 Differential and Integral Calculus 2

Final exam, 21.02.2022 at 9.00–13.00

There are in total 5 questions. Any materials may be used during the exam. Communication with anybody about the exam is strictly forbidden. All work must be your own. Answers must always be justified with calculations and explanations. In the sketches of graphs or pictures, in order to help us understand your sketch, indicate in the picture where is your function, graph, curve, surface.

On the grading:

1. *If you have not done any assignments, then 100% of your final grade comes from the marks you get from this exam (from all 5 questions)*
2. *If you have assignment/exercise marks (like most students taking this exam), there are two ways your final grade is determined:*
   - *(a) Firstly, we will look at 4 of your answers that earn the most points out of all 5 and this will determine 40% of your grade. The rest 60% will come from the assignment/exercise marks done during the course.*
   - *(b) Secondly, we will grade you as we would in case 1, that is, we will also look at what your mark would be if all 5 (including the answer with the lowest mark) are graded and what mark you would get if we do not take assignment/exercise marks into account.*

   *Then the **better** (max) of these two performances in (a) and (b) will determine your final grade. Therefore, I recommend to attempt all 5, even writing down some ideas on them if you are unsure how to start.*

Question 1. a) (3pts) Compute the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x+y}$$

b) (4pts) Let $f(x,y) = -x^2 - y^2$. Sketch the surface the graph of f forms. Find the gradient $\nabla f(x,y)$. Find a point $(x,y) \in \mathbb{R}^2$ such that $\nabla f(x,y) = \langle x, y\rangle$.

c) (3pts) Compute the double integral

$$\int_0^1 \int_0^y e^{x+y}\, dx\, dy.$$

Question 2. Let Γ be the curve with parametric equation $\overrightarrow{\mathbf{r}}(t) = \langle t, 4 + 2t^2 \rangle$, for $0 \leq t \leq 1$.

a) (4pts) Sketch the curve Γ.

b) (3pts) Find the derivative vector $\overrightarrow{\mathbf{r}}'(\frac{1}{2})$.

c) (3pts) Write an integral (purely in terms of t) for the arc length of the curve Γ. You do NOT have to evaluate the integral.

Question 3. Consider the function $f(x, y) = x^3 - 2xy^2 - 7y - 1$.

a) (3pts) Compute all the 1st and 2nd order derivatives.

b) (4pts) Find all the points on the surface where the tangent plane is horizontal.

c) (3pts) Find the tangent plane to the surface at the point $(1, 1)$.

Question 4. a) (5pts) Find the absolute extrema (ie min and max) of $f(x, y) = xy + 3$ on the ellipse $x^2 + 3y^2 \leq 10$.

b) (5pts) Find the center of mass of a two-dimensional plate that occupies the region enclosed by $x = y^2$, $y = 2x^2$ and has density function $\rho(x, y) = xy$. Sketch the plate and its center of mass.

Question 5. a) Consider the double integral $\int_0^4 \int_{\sqrt{x}}^2 \cos(y^3)\, dy\, dx$

i) (2pts) Sketch the region of integration.

ii) (2pts) Write an equivalent iterated integral with the order of integration reversed.

iii) (2pts) Evaluate the integral.

b) (4pts) Find the area of that part of the plane $z = 2x$ that lies inside the paraboloid $z = x^2 + y^2$.