

Instructions: Write briefly and clearly, but give reasons and intermediate results in your answers. A number only as an answer does not yield points. The exam has 4 problems, each worth 6 points.

Write your answers in clear handwriting, either on paper or a tablet computer, and send them in PDF form to the return box on the exam page. Make sure your answers contain: course code, last name, first name, student number and date.

At the end of the exam sheet there is a table for the CDF of the standard normal distribution.

P1 Ville and Kalle ride separate buses to the city center in order to meet. Their buses start at the same time. Ville's travel time has normal distribution with mean 20 minutes and standard deviation 3 minutes. Kalle's travel time has normal distribution with mean 22 minutes and standard deviation 3 minutes. The travel times are independent. Let X denote how much Kalle arrives after Ville (negative if Kalle arrives first).

- (a) What is the distribution of X ? (Give the name and parameters.) Why? **(2p)**
- (b) What is the probability that Kalle arrives first? **(2p)**
- (c) What is the probability that one of them has to wait more than 3 minutes? **(2p)**

P2 The engine of a spaceship runs for a time T (seconds) and it causes the acceleration $a = 2 \text{ m/s}^2$. The distance travelled during acceleration is $X = \frac{1}{2}aT^2$. The time T is a continuous random variable uniformly distributed in the interval $I = [100, 150 + r]$ (seconds), where r is the **first digit** of your student number (one of $0, \dots, 9$, ignore possible letters). Write the value of r clearly in your answer and use this value in your calculations.

- (a) Calculate the expected value of the distance travelled. **(2p)**
- (b) Calculate the probability that the distance is more than 20 000 meters. **(2p)**
- (c) Calculate the standard deviation of the distance. **(1p)**
- (d) Ten similar spaceships are sent, and their engines are run for times T_1, \dots, T_{10} , which are independent and uniformly distributed over the same interval I . Calculate the probability that exactly three of the ships travel more than 20 000 meters during the acceleration. **(1p)**

P3 60% of Finns are “young” (less than 50 years) and the rest are “old”. 35% of the young and 85% of the old use eyeglasses. From the population we take a random sample of 100 persons with replacement. The random variable X_i is 1 if the i th person in the sample is old, otherwise 0. The random variable Y_i is 1 if the i th person uses eyeglasses, otherwise 0. Furthermore X is the number of old people in the sample and Y is the number of people using eyeglasses in the sample.

- (a) Calculate $\mathbb{E}(X_1)$ and $\mathbb{E}(X)$. **(1p)**
- (b) Calculate $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y)$. **(2p)**
- (c) Calculate $\text{Cov}(X_1, Y_1)$. **(1p)**
- (d) Calculate $\text{Cov}(X_1, Y_2)$. **(1p)**
- (e) Calculate $\text{Cov}(X, Y)$. **(1p)**

P4 An exam in Skepticism contains six questions. In each question the student has to choose one of two possible answers. Each correct answer yields one point. $2/3$ of the participants are hard-working and the remaining ones are lazy. A hard-working student answers each question correctly with probability 0.9, independent of the other questions. A lazy student chooses each answer uniformly at random from the two choices and gets it right with probability 0.5.

- (a) If a student is hard-working, what is the probability that he/she gets exactly 3 points?
(2p)
- (b) If a student is lazy, what is the probability that he/she gets exactly 3 points? **(2p)**
- (c) If a student gets exactly 3 points, what is the probability that he/she is lazy? **(2p)**

Normal distribution table

The table below contains numerical values of the standard normal cumulative distribution function

$$\Phi(x) = F_Z(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999