PHYS-C0256 - Thermodynamics and Statistical Physics
Exam on Feb. 23, 2022

6 problems - 30 points

1. Which combination of the following quantities defines a thermodynamic potential? Which potential is it? (1 point)
(i) T, p, N
(ii) T, p, μ
(iii) S, p, N

2. **(i)** For ideal gas show that $C_p - C_V = VT\beta_p^2/\kappa_T$, where β_p and κ_T are isobaric expansivity and isothermal compressibility, respectively. (2 points)
(ii) For a van der Waals gas

$$(p + \frac{a}{V^2})(V - b) = RT \tag{1}$$

where a and b are constant, show that in the limit of $V \to \infty$ at constant p, one obtains the ideal gas result for $C_p - C_V$. (Hint: you can use $C_p - C_V = [p + (\frac{\partial U}{\partial V})_T](\frac{\partial V}{\partial T})_p$.) (2 points)
(iii) A gas obeys the following equation of state,

$$p(V - Nb) = Nk_BT \exp{(-\frac{a}{V - Nb})}, \tag{2}$$

where a and b are the parameters of the system and N is kept fixed. Use a differential for S to calculate $dU(T, V)$ for the internal energy U. Show that for the given equation of state U is a function of T only. (2 points)

3. The body with heat capacity C is connected by thermal conductance G_{th} to the environment that has a fixed temperature T_0, see Fig. 1. Initially the power applied to the body is $\dot{Q} = 0$, and its temperature T is that of the environment. At time $t = 0$ a constant power $\dot{Q} = \dot{Q}_0$ is switched on. Find the time dependent temperature of the body at $t > 0$ assuming that both C and G_{th} do not depend on temperature. (Hint: a similar problem of a lamp was analysed in the exercise session) (4 points)

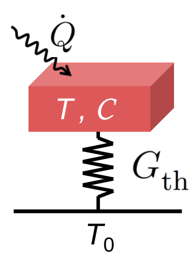

FIG. 1. Setup for problem 3.

4. Consider a four-level system in form of a paramagnet with energy levels $E = -m\bar{\mu}B$, where m assumes values $-3/2$, $-1/2$, $+1/2$ and $+3/2$. B is the magnetic field and $\bar{\mu}$ the effective magnetic moment. Find for a system of N independent such four-level systems:
(i) the partition function Z (2 points)
(ii) the entropy S (2 points)
(iii) and the heat capacity C_B at a constant field B. (2 points)
(iv) Write down the time dependent temperature of this system when it is heated with power P assuming it is at temperature T_0 initially. (3 points)

5. Show that for normal metal-insulator-superconductor NIS junction at low temperature T, when $k_BT \ll \Delta$, the following equation holds approximately for its cooling power of N, $\dot{Q}_{\text{NIS}} = 0.59\frac{\Delta^2}{e^2R_T}(k_BT/\Delta)^{3/2}$, at $V \simeq \Delta/e$. A similar calculation shows that the current at this point is approximately $I = 0.48\frac{\Delta}{eR_T}\sqrt{k_BT/\Delta}$. Write the efficiency η of the NIS refrigerator at this optimum point. (4 points)

6. A quantum two-level system (qubit) with energy level separation E is coupled to a heat bath. This coupling gives the transition rates $\Gamma_\uparrow$ from ground state to the excited state, and $\Gamma_\downarrow$ from the excited state to the ground state.

(i) The qubit is reset to the ground state by an external pulse at the initial time instant $t = 0$. Find the time dependent population of the qubit in the excited state ρ_{ee} as a function of time. What can you say about the off-diagonal elements of the density matrix in this process? (3 points)

(ii) If the resetting of the qubit is repeated periodically with time intervals Δt, find the average power (over a long period of time) that this refrigerator draws from the bath. (Hint: instantaneous power that the qubit draws from the bath that it is connected to, is given by $Ed\rho_{ee}/dt$.) (3 points)